

NAME:.....

Section:.....

MATH 151 (Fall 2008) Dr. Z. , **First Practice For First Midterm Exam**

1. (12 points [3 pts each]) Find the limits

$$(a) \quad \lim_{x \rightarrow 2} \frac{4x + 2}{x + 2}$$

$$(b) \quad \lim_{x \rightarrow 1} \frac{\sqrt{2x + 2} - 4}{x - 1}$$

$$(c) \quad \lim_{x \rightarrow 0} \frac{15 - 15 \cos x + 3 \sin x + x}{x}$$

$$(d) \quad \lim_{x \rightarrow 0} \frac{\sin^7 2x}{x^2 \sin^3 2x \sin^2 4x}$$

2. (12 points) Find $\frac{dy}{dx}$ by implicit differentiation if

$$x^3y + 3xy^2 + xy^5 = 1000 \quad .$$

3.

(a) (6 points) Show that the equation $2x^{100} = 1$ has a solution in the open interval $0 < x < 1$.

(b) (6 points) Use the **formal definition** of the limit to prove that

$$\lim_{x \rightarrow 10} 5x - 1 = 49 \quad .$$

4. (12 points) Find the points on the curve

$$y = x^5$$

where the tangent line is parallel to the line $y = 5x + 14$. Then find the equations of these tangent lines for each of these points.

5.

(a) (6 points) If $f(x) = \cos x^2 + x^4$, find the second derivative $f''(x)$.

(b) (6 points) If the law of motion is $s = -t^3 + 2t$, find the displacement (position), velocity, and acceleration at $t = 2$. Is it moving forward or backwards then? Is it speeding up or slowing down then?

6. (a) [6 points] Plot the function

$$f(x) = \begin{cases} 2x + 1, & \text{if } -\infty < x < 1; \\ 5, & \text{if } 1 \leq x \leq 2; \\ 3x, & \text{if } 2 < x < \infty; \end{cases}$$

(b) [6 points] Plot the derivative $f'(x)$. Indicate points of non-differentiability by empty dots.

7. (16 points) Find the values of the constants a and b that will make the function

$$f(x) = \begin{cases} x^3, & \text{if } -\infty < x < 1; \\ ax + b, & \text{if } 1 \leq x \leq 2; \\ 5, & \text{if } 2 < x < \infty; \end{cases}$$

continuous everywhere.

8. Using the information

$$\lim_{x \rightarrow 1} f(x) = 2 \quad , \quad \lim_{x \rightarrow 1} g(x) = 3 \quad , \quad \lim_{x \rightarrow 1} h(x) = -1 \quad ,$$

Compute the following limits [3 points each]

(a) $\lim_{x \rightarrow 1} 5f(x) - 3g(x) + h(x)$

(b) $\lim_{x \rightarrow 1} \frac{f(x)g(x)}{h(x)}$

(c) $\lim_{x \rightarrow 1} f(x)^3 g(x)^2 - 2h(x)$

(d) $\lim_{x \rightarrow 1} f(x)^{g(x)}$