

NAME:.....

Section:.....

MATH 151 (Fall 2008) Dr. Z. , **Third Practice For First Midterm Exam**

1. (12 points) Find $f'(-1)$, if

$$f(x) = \frac{2}{\sqrt{2+x}}$$

using the **definition** of the derivative. [No Credit for other methods].

2. A slingshot is used to shoot a pebble in the air vertically from the top of a tower of height 100 meters, with initial velocity 10 meters per second.

(a) [6 points] Write a formula for $s(t)$, the height after t seconds, valid during its flight (ignoring air-resistance)

(b) [3 points] Write a formula for $v(t)$, the velocity of the pebble after t seconds valid during its flight (ignoring air-resistance).

(c) [3 points] What are the height, velocity, and acceleration after 2 seconds.

3. (a) [6 points] Using the **definition** of the limit, prove rigorously that

$$\lim_{x \rightarrow 1} 100x - 50 = 50 \quad .$$

(b) [6 points] Prove that the equation

$$2 \cos x - \sin x = \frac{1}{2}$$

has a solution in the open interval $\pi/4 < x < \pi$.

4. (15 points ([5 pts each]) Find the derivative $f'(x)$ if:

(a) $f(x) = \frac{\sin x}{3x+4}$

(b) $f(x) = x \cos 3x$

(c) $f(x) = \frac{e^{x^2}}{1+3x^2}$

5. (13 points) Find an equation of the tangent line to the curve

$$y = 2x^5 + 3x + 1 \quad ,$$

at the point for which $x = 1$.

6. [12 points] Differentiate the equation with respect to t to calculate $\frac{dy}{dt}$ in terms of $\frac{dx}{dt}$.

$$y^3 + 3xy + x^4 = 11$$

7. [12 points] Find all the points on the graph $3x^2 + 4y^2 + 3xy = 24$ where the tangent line is horizontal.

8. (12 points) Find the point on the curve

$$y = 1 + \sin x \quad ,$$

with $0 < x < \pi$, where the tangent line is parallel to the line $y = \frac{\sqrt{2}x}{2} + 4$. Then find the equation of that tangent line.