Answers to Dr. Z's Three Practice Tests For The Second Midterm Exam
(First Posted Nov. 16, 2008. Version of Nov. 17, 2008 (one mistake corrected, thanks to Sammy G.).

Answers to First Practice Exam for Exam II

(http://www.math.rutgers.edu/˜zeilberg/calc1N/pII1.pdf)

1. (a) Local maximum: $(0,0)$; local minimum $(3,-27)$; (b) $(\frac{3}{2},-\frac{27}{2})$; (c) (i) $(-\infty,0)$, and $(3,\infty)$; (ii) $(0,3)$; (iii) $(\frac{3}{2},\infty)$ (iv) $(-\infty,\frac{3}{2})$. (d) Verbal description: (**you** have to actually sketch it!). It emerges from $(-\infty,-\infty)$, and climbs its way up until it peaks at the local max $(0,0)$. Then it starts sliding down to the local min $(3,-27)$, after which it climbs up for ever after to (∞,∞). At $(\frac{3}{2},-\frac{27}{2})$ (half-way in the descent from $(0,0)$ to $(3,-27)$) there is a point of inflection, where it ceases to be concave down, and starts being concave up).

2. (a) $125000/\sqrt{63536}$ (= 499.856..., but you need a calculator for that). (b) 0.

3. $(1,1)$.

4. (a) $\frac{9}{4}$; (b) Since $f''(x)=(-1/4)(1-x)^{-3/2}$, and $f''(-4)<0$, the function is concave down at $x=-4$, so the estimate is **larger** then the actual value.

5. (a) $(-\sin x+\cos x)\ln(\tan^{-1}x)+\frac{\cos x+\sin x}{(x^2+1)\tan^{-1}(x)}$; (b) $\frac{1}{(\ln 2)(1+x^2)\tan^{-1}x}$; (c) $-\sin x$.

6. (a): concave up: $(-\infty,-2-\sqrt{5})$ and $(-2+\sqrt{5},\infty)$; concave down: $(-2-\sqrt{5},-2+\sqrt{5})$; p.o.i: $x=-2-\sqrt{5}$ and $x=-2+\sqrt{5}$. (b) concave down: $(-\infty,\frac{3}{5})$;concave up: $(\frac{3}{5},\infty)$; p.o.i. $x=\frac{3}{5}$. (Remark: $x=0$ is **not** a p.o.i. since $f''(x)$ does not change sign there. It was a false alarm).

7. (a)

$$\left(\frac{10}{x(1+\ln x)}+\frac{20e^x}{3+e^x}-\frac{50}{\sqrt{x}-x}\right)(1+\ln x)^{10}(3+e^x)^{20}(1-\sqrt{x})^{100}$$

(b) $y=\cos x+x^2+1$.

8. (a) $\frac{1}{4}x^4+x^2+C$; (b) $\frac{1}{2}x^2+C$; (c) $\frac{1}{4}x^4-\frac{1}{3}x^3+C$.

Answers to Second Practice Exam for Exam II

(http://www.math.rutgers.edu/ ˜zeilberg/calc1N/pII2.pdf)

1. (a) $y=0$ (on the left and on the right); (b) None ; (c) Local minimum: $(-1,-\frac{1}{2})$, Local maximum: $(1,\frac{1}{2})$; (d) $(-\sqrt{3},-\sqrt{3}/4)$, $(0,0)$, $(\sqrt{3},\sqrt{3}/4)$; (e) (i) $(-1,1)$; (ii) $(-\infty,-1)$,$(1,\infty)$; (iii) $(-\sqrt{3},0)$, $(\sqrt{3},0)$; (iv) $(-\infty,-\sqrt{3})$, $(0,\sqrt{3})$. (f) Verbal description: (**you** have to actually sketch it!). It comes from $(-\infty,0)$, barely under the negative x-axis (that is a horiz. asymptote), passing through the point of inflection $(-\sqrt{3},-\sqrt{3}/4)$, where it ceases to be concave down and starts being concave up, reaching the local minimum $(-1,-\frac{1}{2})$. Now it starts to climb, passing exactly through the origin, that is a point of inflection, where is stops being concave down, and starts being concave up, until it reaches the local maximum $(1, \frac{1}{2})$. After that it is for ever after decreasing, first passing through the last point of inflection $(\sqrt{3}, \sqrt{3}/4)$, where it ceases to be concave down and starts being concave up (for ever after), eventually going just-above the positive x-axis, but never actually making it there (well it does at $x = \infty$, but that will never happen in our lifetime).

2. (a) $\frac{4}{11}$; (b) $\frac{1}{24}$;

3. (a) $x_2 = \frac{1}{3}$; (b) $\frac{50e^5+1}{10e^5}$.

4. (a) $\frac{1}{10}$ cm/s ; (b) $\frac{1}{8} - \frac{3}{16}(x-2)$.

5. (a) $3x^2(\ln(1+x))^2 + \frac{2x^3\ln(1+x)}{1+x}$; (b) $\frac{(\ln 10)10^{\tan^{-1}x}}{1+x^2}$; (c) $3x^2$.

6. $10 \times 10 \times \frac{25}{3}$.

7.(a) Abs. max.: 8 (at $x = 8$); abs. min: -1 (at $x = 1$). (b) $y = \tan x + \frac{1}{2}x^2 + 2$.

8. 4 (you had to do it the **hard** way).

Answers to Third Practice Exam for Exam II

(http://www.math.rutgers.edu/˜zeilberg/calc1N/pII3.pdf)

1. (a) H.A. $y = \frac{1}{2}$; V.A.: $x = \frac{1}{2}$. (b) None; (c) None; (d) (i) Nowhere ; (ii) $(-\infty, \frac{1}{2})$,$(\frac{1}{2}, \infty)$; (iii)$(\frac{1}{2}, \infty)$; (iv)$(-\infty, \frac{1}{2})$; (e) Verbal description: (**you** have to actually sketch it!). It comes from $(-\infty, \frac{1}{2})$, practically going parallel to the horiz. asymptote $y = \frac{1}{2}$, just a teeny-weeny under it. Eventually it starts climbing down, passing through the origin, and then it dives down to $(\frac{1}{2}, -\infty)$, only to emerge immediately from $(\frac{1}{2}, \infty)$, starting its descent down to $(\infty, \frac{1}{2})$, eventually going practically parallel to the horiz, asymptote $y = \frac{1}{2}$, barely above it, but never actually making it during our lifetime.

2. (a) -6 ; (b) $\frac{3}{4}$; (c) DNE (∞ is also an acceptable answer)

3. $\frac{7}{5}$ ohms per sec.

4. (a) $h = \frac{x}{9}$ (b) $\frac{2}{3}$ meters, $\frac{1}{9}$ m/s.

5. $200\sqrt{6}$ ft.

6. $\frac{4\sqrt{3}}{9}$.

7. (a)

$$\frac{100(1+\ln x)^{99}(\sin x + \cos x)^{50}}{x} + 50(1+\ln x)^{100}(\sin x + \cos x)^{49}(\cos x - \sin x) \quad ;$$

(b) $y(x) = x^3$.

8. (a) 20; (b) 12; (c) R_4, since the region whose area is R_4 **contains** the region under $y = 2x - 2$ and above the x-axis (between $x = 1$ and $x = 5$), whereas the the region whose area is L_4 **is contained** in that region.