Answers to Dr. Z's Three Practice Final Exams
Disclaimer: Not repsponsible for any typos or other erros. Award of $1 for any mistake found).
Note: Sometimes there is more than one "correct" answer, but using algebra, one should be able to go from one correct answer to another.

Answers to First Practice Final
(http://www.math.rutgers.edu/~zeilberg/calc1N/pf1.pdf)

1. $\frac{-1}{2\sqrt{x+1}^3}$ (but you had to do it from the definition!)
2.
$$\frac{-y^4-3x^2y-y+8x}{x(x^2+4y^3+1)}$$

3. (a) 9, forward; (b) $t=-1, t=1$; (c) 22.
4. (a)
$$-\frac{(x+1)\sin x+\cos x}{(x+1)^2}$$

(b) $\sin x\cos x + x(\cos^2 x - \sin^2 x)$ OR $\frac{1}{2}\sin 2x + x\cos 2x$
(c)
$$\frac{3x\cos x-4-4\sin x}{(x-\cos x)^2}$$

(d)
$$\frac{e^{2x}(1+2x)}{(1+x)^2} \quad .$$

5. $y=2x+2$
6. (a) -4 (b) $\frac{1}{2}$ (c) 1 (d) 3
7. (a) $3, 0, DNE, 9, -3, DNE$ (b) $x=0, x=3$
8. Point: $(2,0)$. Eq. of tangnet line: $y=x-2$
9. (a)
$$\frac{6(1-3x^4)}{(x^4+1)^2}$$

(b) $5(5t-2)e^{-5t}$
(c) $(2-x^2)\sin x + 4x\cos x$.
10. H.A.: $y=0$; V.A: $x=-1$, $x=1$; Local max: none; Local Min: none; Point of Inflection: $(0,0)$; increasing: none; decreasing: $(-\infty,-1),(-1,1),(1,\infty)$. Verbal description: at the extreme left barely touches the x-axis, just a tiny bit below, eventually, as it reaches $x=-1$ shoots down to $-\infty$, only to emerge, right after $x=-1$ from ∞, going down, via the origin, that it the only point of inflection, down down to $-\infty$ at $x=1$, at $x=1$

it suddenly jumps to ∞ and starts its descent down, eventually getting very close, and almost parallel to the x-axis, but never crossing it.

11. $\frac{41}{49}\Omega/s$.

12. (a)$1 - \frac{1}{100e}$ (b) $\frac{5}{3}$

13. (a) $\sqrt[3]{2}$ (b) abs. max: 9 at $x = 2$; abs. min: -2 at $x = 1$.

14. inc.: $(-\infty, 1)$, dec.: $(1, \infty)$, (b) local max: $(1, \frac{1}{e})$, local min: none, (c) concave down: $(-\infty, 2)$, concave up: $(2, \infty)$, point of inflection: $(2, \frac{2}{e^2})$. (d) Verbal description: comes from the left from $-\infty$, climbing up, passing through the origin, peaking at the max. $(1, \frac{1}{e})$, where it goes down, passing through the inflection point $(2, \frac{2}{e^2})$, and keep going down, but slower and slower, eventually barely touching the positive t-axis.

15. $10 \times 10 \times 10$.

16. a) $\frac{3}{2}x^2 + 5\sin^{-1} x + C$ (b) $\frac{x^2}{2} + x + \ln|x| + C$ (c) $x^2 + \frac{1}{x^3} + 1$.

Answers to Second Practice Final

(http://www.math.rutgers.edu/~zeilberg/calc1N/pf2.pdf)

1. -3 (but you had to do it from the definition!)

2. (a) 2 (b) 2

3.(a) $c = \frac{1}{4}$ (b) $c = \frac{1}{2}$, $d = -\frac{1}{2}$.

4. (a) $\frac{49}{10}$; (b) larger (because curve is concave down at $x = 20$).

5. (a) 0 (b) 0 (c) 2

6. (a) abs. max.: 8 at $x = -8$; abs. min: -1 at $x = 1$. (b) abs. min.: 0 at $x = 0$; abs. max: $\frac{1}{e}$ at $x = 1$

7. (a) 2 by 6 (b) 2

8. (a) $y = \frac{4}{3}x^3 + 2x$ (b) $y = -2\sin x - \cos x - x + 3 + \frac{\pi}{2}$

9. a) H.A.: $y = 0$, V.A.: $x = -3, x = 3$. b) dec.: $(-\infty, -2), (-2, 0)$; inc.: $(0, 2), (2, \infty)$. local min:$(0, \frac{4}{9})$, local max: none. c): concave up: $(-3, 3)$, concave down: $(-\infty, -3)$,$(3, \infty)$, points of inflection: none. d): you do it!

10.(a) $y = \frac{2}{9}x + \frac{5}{9}$; (b) $y = -\frac{3}{2}x + 6$.

11. (a) 40 (b) Use IVT, $f(-4)$ is neg. while $f(3)$ is pos.

12. (a)

$$\frac{\frac{-3}{x^2}e^{\frac{3}{x}}\cos(x^2 - x) + e^{\frac{3}{x}}(2x - 1)\sin(x^2 - x)}{\cos^2(x^2 - x)}$$

(b)

$$\frac{3}{2}\frac{\sqrt{\sin^{-1} x}(2\ln x - \ln(x + 1))}{\sqrt{1 - x^2}} + (\sin^{-1} x)^{3/2}\left(\frac{2}{x} - \frac{1}{x + 1}\right)$$

(c) $2xe^{x^2}\cos(e^{x^2})$

13. (a) $\frac{3}{4}$ (b)$\frac{1}{6}$ (c) 0 (the integrand is an odd function [why?]).

14. (a)$h = \frac{4000}{x}$ (b) height of shadow: 25 meters; rate of change of height of shadow: $-\frac{5}{8}$ m/sec. (c) $-\frac{1}{325}$ radians per sec.

15. (a) $-\frac{1}{4}e^{\frac{2}{x^2}} + C$ (b) $\pi^4\sqrt{\pi}/2304 + \frac{1}{2}$ (c) 50π (d) $x = 0$ and $x = -1$.

Answers to Third Practice Final

(http://www.math.rutgers.edu/~zeilberg/calc1N/pf3.pdf)

1. $-\frac{1}{4}$ (but you had to do it from the definition).

2. (a) 20 (b) $\frac{7}{3}$

3. (a) $\frac{x}{2} + \frac{\sin 2x}{4} + C$, $\frac{x}{2} - \frac{\sin 2x}{4} + C$ (b)$\frac{319}{420}, \frac{4448}{6435}, \frac{533}{840}$.

4. (a) $\frac{1}{2}n(6n^2 + 3n - 1)$ (b) use symmetry.

5. (a) $\frac{1}{4}\tan^4 x + C$ (b) $\sqrt{2 + x^3}, -\sqrt{2 + x^3}$ (c) $2xe^{x^6} - e^{x^3}$

6. (a) $\frac{1}{2}$ (b) 1.

7. $(2 + \sqrt{2}, \sqrt{\ln(2 + \sqrt{2})})$

8. 200 by 80.

9. (a) inc.: $(-\infty, 0)$, $(1, \infty)$; dec.: $(0, 1)$. Local max: $(0, 1)$, local min: $(1, 0)$. (b) concave up: $(\frac{1}{2}, \infty)$; concave down: $(-\infty, \frac{1}{2})$. Point of inflection: $(\frac{1}{2}, \frac{1}{2})$ (c) You do it!

10. inc.: $(e^{-1/3}, \infty)$; dec.: $(-\infty, e^{-1/3})$. Local min: $(e^{-1/3}, -\frac{1}{3e})$,
concave down: $(-\infty, e^{-5/6})$, concave up: $(e^{-5/6}, \infty)$; p.o.i: $(e^{-5/6}, -\frac{5}{6}e^{-5/2})$.

11. (a) $f(5) \leq -9$ (b) $y = 4, y = -4$ (c) $y = 2, y = -2$.

12. (a)

$$\frac{4x^3}{1 + (x^4 + 1)^2}$$

(b)

$$\frac{5x^4 + 2e^{2x}}{(\ln 10)(x^5 + e^{2x})}$$

(c)

$$(x^2 + 4)^{-1/2} - x^2(x^2 + 4)^{-3/2}$$

13.(a) $-\frac{4}{5}$ (b)

$$-\frac{2y^3 + 2xy^2 + 2y + 3x^2}{6xy^2 + 2x^2y + 2x + 1}$$

14. $\frac{1}{300}$ m/s.

15. (a) abs. max: 9 and $x = 3$; abs. min: 4 at $x = 2$. (b) H.A.: $y = -1$, V.A.: $x = -1, x = 1, x = -2, x = 2$.