Solutions to the "QUIZ" for Dec. 1, 2008

1. Compute :

$$\int_0^3 \frac{dx}{x^2+9} \quad .$$

Sol. to 1: We make the substitution $x = 3u$. Then $dx = 3du$ and don't forger to translate the limits! When $x = 0$, $u = 0$, when $x = 3$, $u = 1$. We have:

$$\int_0^3 \frac{dx}{x^2+9} = \int_0^1 \frac{3du}{(3u)^2+9} =$$

$$\int_0^1 \frac{3du}{9u^2+9} = \int_0^1 \frac{3du}{9(u^2+1)} = \frac{1}{3}\int_0^1 \frac{du}{(u^2+1)} = \frac{1}{3}\left(\tan^{-1} u \Big|_0^1\right) =$$

$$\frac{1}{3}(\tan^{-1} 1 - \tan^{-1} 0) = \frac{1}{3}(\frac{\pi}{4} - 0) = \frac{\pi}{12} \quad .$$

Ans. to 1: $\frac{\pi}{12}$.

Comments: Many people forgot to transform the limits of integration (0 and 3) from the x-language to the u-language.

2. Compute

$$\int_0^2 5^x \, dx \quad .$$

Sol. to 2:

The easiest way is to use the formula

$$\int a^x \, dx = \frac{a^x}{\ln a} + C \quad .$$

We have:

$$\int_0^2 5^x = \frac{5^x}{\ln 5}\Big|_0^2 = \frac{5^2}{\ln 5} - \frac{5^0}{\ln 5} =$$

$$\frac{25}{\ln 5} - \frac{1}{\ln 5} = \frac{24}{\ln 5} \quad .$$

Ans. to 2): $\frac{24}{\ln 5}$.

Comments: Many people **multiplied** by $\ln 5$ instead of **dividing**. When you **differentiate**, you multiply:

$$(5^x)' = (\ln 5)5^x \quad ,$$

But when you **anti-differentiate** you divide:

$$\int 5^x \, dx = \frac{5^x}{\ln 5} + C \quad .$$

A few people did:

$$\int 5^x \, dx = \frac{5^{x+1}}{x+1} + C \quad .$$

This is **wrong wrong wrong**. Don't confuse

$$\int x^5 \, dx \quad ,$$

where the x is at the base and the power is 5 with

$$\int 5^x \, dx$$

where the base is the number 5 and the power (exponent) is x.