Solutions to the "QUIZ" for Dec. 4, 2008

1. Find the area of the region enclosed between $y = x^2 - 2x$ and $y = x - 2$.

Sol. of 1: We first find the **points of intersection** by setting them equal to each other

$$x^2 - 2x = x - 2 \quad ,$$

means

$$x^2 - 3x + 2 = 0 \quad .$$

Factoring, gives

$$(x-1)(x-2) = 0 \quad ,$$

so we have **two** solutions $x = 1$ and $x = 2$. So the limits of integration are $x = 1$ and $x = 2$, and the area is

$$\int_1^2 (TOP(x) - BOT(x))\, dx$$

(Since we only got two solutions, there is only one interval, so we don't have to split the area-computation into several intervals).

It remains to see who is on top and who is on bottom.

Plugging-in $x = 1.5$ into $y = x^2 - 2x$ gives $2.25 - 3 = -0.75$, Plugging-in $x = 1.5$ into $y = x - 2$ gives $1.5 - 2 = -0.5$. Since -0.5 is **larger** than -0.75, it follows that $TOP = x - 2$ and $BOT = x^2 - 2x$.

So the area is:

$$\int_1^2 (x - 2 - (x^2 - 2x))\, dx = \int_1^2 (-x^2 + 3x - 2)\, dx = \frac{-x^3}{3} + \frac{3x^2}{2} - 2x \Big|_1^2$$

$$= [-\frac{(2)^3}{3} + \frac{3 \cdot 2^2}{2} - 2 \cdot 2] - [-\frac{(1)^3}{3} + \frac{3 \cdot 1^2}{2} - 2 \cdot 1]$$

$$= [-\frac{8}{3} + 6 - 4] - [-\frac{1}{3} + \frac{3}{2} - 2] = [-\frac{8}{3} + 2] - [-\frac{1}{3} + \frac{3}{2} - 2] = -\frac{2}{3} + [\frac{1}{3} + \frac{1}{2}] = -\frac{2}{3} + \frac{5}{6} = \frac{1}{6} \quad .$$

Ans. to 1: The area is $\frac{1}{6}$.

Comments: Almost everyone got the right set-up, and the fact that the limits of integration are 1 and 2. Only about %40 got the right answer. Many people messed up the arithmetics, and some people got the TOP and BOT revrersed, because they messed up in the plugging-in stage (of $x = 1.5$).