Solutions to the "QUIZ" for Nov. 3, 2008

1. A farmer has 3000 meters of fence, and needs to make a rectangular animal-shed, with a partition in the middle, parallel to one of the sides, using the same fencing material. What are the dimensions of the rectangle that would maximize the area?

Solution to 1: Let's call the side parallel to the parition x, and the other side y.

The constraint is

$$3x + 2y = 3000$$

and the Goal function is the area xy. Solving for y, in terms of x, we get

$$2y = 3000 - 3x \quad ,$$

and dividing by 3, we get

$$y = 1500 - \frac{3}{2}x \quad .$$

Plugging this into the goal function xy, we get, that the goal function, in terms of x alone is:

$$f(x) = x(1500 - \frac{3}{2}x) = 1500x - \frac{3}{2}x^2 \quad .$$

To find the **maximum**, we take the derivative

$$f'(x) = 1500 - 3x$$

and set it equal to 0.

$$1500 - 3x = 0 \quad .$$

Solving we get $x = 500$ and $y = 1500 - \frac{3}{2}500 = 750$.

Ans. to 1: The dimensions of the rectangle that maximize the area are 500×750.

Comments: Many people set up the constraint at $xy = 3000$. In other words, they literally aped the problem in the handout that we did in class. In that problem, the area was **fixed**, and we had to **minimize** the fencing. In the present problem, the **fencing** is fixed, so the constraint is "amount of fence=3000". Setting $area = 3000$ is a **big** conceptual mistake. You were told that the amount of fencing is 3000 meters (units of lenght), so $area = 3000$ does not make sense, since the units of area are square meters. So **watch out**.

2. Find the point on the curve $y = \sqrt{x}$ that is closet to the point $(1, 0)$.

Solution to 2.

The $(\text{distance})^2$ of any point (x, y) to $(1, 0)$ is:

$$(x-1)^2 + (y-0)^2 = x^2 - 2x + 1 + y^2$$

The **constraint** is $y = \sqrt{x}$, and squaring, $y^2 = x$. Plugging this into the above goal function, we get that the distance-squared, in terms of x alone, let's call it $f(x)$ is:

$$f(x) = x^2 - 2x + 1 + x = x^2 - x + 1 \quad .$$

Now let's minimize it.

$$f'(x) = 2x - 1 \quad .$$

Setting this equal to 0 gives $x = \frac{1}{2}$, and plugging into $y = \sqrt{x}$ we get $x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$.

Ans. to 2: The point on the curve $y = \sqrt{x}$ closest to $(1, 0)$ is $(1/2, \sqrt{2}/2)$.

Comments: Most people got it right, but as usual, some people messed up the algebra, or had the wrong distance formula, or confused $(1, 0)$ with $(0, 1)$.