Solution to the"QUIZ" for Oct. 16, 2008

Question: If three resistors with resistence R_1 and R_2 and R_3 are connected in parallel, then the total resistance is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad .$$

If R_1, R_2, R_3 are increasing at a rate of 1, 2, and 3 ohms per sec. respectively, how fast is R changing when $R_1 = 1$ and $R_2 = 1/2$ and $R_3 = 1/3$.

Solution: For future reference, we would need R:

$$\frac{1}{R} = \frac{1}{1} + \frac{1}{1/2} + \frac{1}{1/3} = 1 + 2 + 3 = 6 \quad ,$$

so $R = \frac{1}{6}$.

Next, let's rewrite the relation in power notation:

$$R^{-1} = R_1^{-1} + R_2^{-1} + R_3^{-1} \quad .$$

Now, differentiate both parts with respect to t:

$$(-1)R^{-2}\frac{dR}{dt} = (-1)R_1^{-2}\frac{dR_1}{dt} + (-1)R_2^{-2}\frac{dR_2}{dt} + (-1)R_3^{-2}\frac{dR_3}{dt} \quad .$$

Now plug-in all the available data:

$$(-1)(1/6)^{-2}\frac{dR}{dt} = (-1)(1)^{-2} \cdot 1 + (-1)(1/2)^{-2} \cdot 2 + (-1)(1/3)^{-2} \cdot 3 \quad .$$

Simplifying:

$$-36\frac{dR}{dt} = -(1 + 8 + 27)$$

yielding

$$\frac{dR}{dt} = 1 \quad .$$

Ans.: R is changing at a rate of 1 ohm per second.

Comments: Most people started it out correctly, doing the differentiations. But only about %40 of the people got the correct final answer. Some people should go back to first-grade: (learn that $1 + 2 + 3$ is equal to 6 **not** 5), and to fourth-grade (or whenever they learn about the advanced concept of fraction), for example that

$$\frac{1}{\frac{1}{6}} = 6 \quad ,$$

and **not** $\frac{1}{6}$, and that $(1/6)^{-2}$ equals 36 and **not** $1/36$.