Solutions to the "QUIZ" for Oct. 2, 2008

1. Use the rules of differentiation to calculate $f'(x)$ (Do not simplify your answer).

$$f(x) = \frac{e^{\frac{1}{x+1}}}{e^x + 11}$$

Sol. 1: **First** use the **quotient rule**:

$$f'(x) = \frac{(e^{\frac{1}{x+1}})'(e^x + 11) - e^{\frac{1}{x+1}}(e^x + 11)'}{(e^x + 11)^2}$$

Since this is rather complicated, it may be **a good idea** to do:

$$\left(e^{\frac{1}{x+1}}\right)'$$

separately. This calls for the **chain rule**:

$$\left(e^{\frac{1}{x+1}}\right)' = (e^{(1+x)^{-1}})' = e^{(1+x)^{-1}}((1+x)^{-1})' = e^{(1+x)^{-1}}((-1)(1+x)^{-2})$$

$$= -e^{(1+x)^{-1}}(1+x)^{-2} \quad .$$

Now we go back to the **main problem** and have

$$f'(x) = \frac{-e^{\frac{1}{1+x}}(1+x)^{-2}(e^x + 11) - e^{\frac{1}{x+1}}e^x}{(e^x + 11)^2}$$

Since you were not asked to simplify, this is the **answer**.

Comments: Only about %30 of the people got it completely right. Remember to do things in order, and if the problem is too complicated, do a subproblem separately, like I did above.

2. Find the equation of the tangent line at the point $(1, 1)$ to the curve defined by the equation

$$x^2 + 3xy + y^2 = 5 \quad .$$

Solution: First we **differentiate both sides**

$$(x^2 + 3xy + y^2)' = 5' \quad .$$

Using straightforward differentiation (for terms only involving x), and the product rule and chain rules, we have

$$(x^2)' + (3xy)' + (y^2)' = 0 \quad .$$

$$2x + (3x)'y + (3x)y' + 2yy' = 0 \quad ,$$

which boils down to:

$$2x + 3y + (3x)y' + 2yy' = 0 \quad ,$$

Now that we have done all the differentiations, it is time to **plug-in** the **specific** values for x and y. Since we are talking about the point $(1,1)$, we have to plug-in $x = 1$ and $y = 1$.

$$2(1) + 3(1) + (3 \cdot 1)y' + 2(1)y' = 0 \quad ,$$

which becomes

$$5 + 5y' = 0 \quad .$$

which means:

$$5y' = -5 \quad ,$$

Dividing both sides by 5 we have

$$y' = -1 \quad .$$

This means that the **slope** of the tangent is $m = -1$. This ends the **first phase**.

We still need to find the **equation of the tangent line**. The **point** is $(x_0, y_0) = (1,1)$ and the **slope** is $m = -1$. Using the famous equation

$$(y - y_0) = m(x - x_0) \quad ,$$

we get

$$(y - 1) = (-1)(x - 1) \quad ,$$

and simplifying, we get

$$y = -x + 2 \quad ,$$

Ans. in English: The equation of the tangent line at the point $(1,1)$ to the curve $x^2 + 3xy + y^2 = 5$ is $y = -x + 2$.

Comments: About %70 got it correctly. Quite a few people messed up the very simple algebra. Some people gave the solution $y' = 1$ (instead of $y' = -1$) to the equation $5y' + 5 = 0$. **Don't trust your head**. Do everything on paper.

Some people used the Leibnitz notation $\frac{dy}{dx}$ instead of y'. That's perfectly OK. y' and $\frac{dy}{dx}$ are exactly the same things, it is just easier to write y'.

Some poeple first found the general expression for y' (i.e. they didn't plug-in $x = 1$, $y = 1$), as soon as possible, and got from

$$2x + 3y + (3x)y' + 2yy' = 0 \quad ,$$

$$2x + 3y + (3x + 2y)y' = 0 \quad ,$$

$$(3x + 2y)y' = -(2x + 3y)$$

and so

$$y' = -\frac{2x + 3y}{3x + 2y} \quad ,$$

(this is what you have to do if you were asked to find $\frac{dy}{dx}$ (alias y') and were not given a specific point.) These people plugged-in $x = 1$ and $y = 1$ at the very end, getting (correctly!)

$$y'\Big|_{x=1,y=1} = -\frac{2(1) + 3(1)}{3(1) + 2(1)} = -1 \quad .$$

This problem is simple enough that there is no much advantage to do it the previous way, but for more complicated problems, it is more efficient to do the plugging-in of x and y as soon as you can, and solve for y' in the numerical mode.