Solutions to the "QUIZ" for Oct. 20, 2008

1. Let $f(x) = \sqrt{4+x}$. Using the linear approximation of $f(x)$ at $a = 5$ compute an approximation to $f(4)$.

First Solution of 1: The **Linearization** of $f(x)$ at $x = a$, let's call it $L(x)$, is, in general

$$L(x) = f(a) + f'(a)(x-a) \quad .$$

In this problem $f(x) = (4+x)^{1/2}$, and $a = 5$. We first need $f'(x)$:

$$f'(x) = (1/2)(4+x)^{-1/2} = \frac{1}{2\sqrt{4+x}} \quad .$$

plugging-in $x = 5$ we get:

$$f'(5) = \frac{1}{2\sqrt{4+5}} = \frac{1}{2\sqrt{9}} = \frac{1}{6} \quad .$$

Also $f(5) = \sqrt{9} = 3$.

Plugging-in $a = 5$ and $f'(5) = 1/6$ into the general formula we have

$$L(x) = 3 + \frac{1}{6}(x-5) \quad .$$

This is the **linearlization**. To get the **linear approximation** for $f(4)$, we plug-in $x = 4$

$$L(4) = 3 + \frac{1}{6}(4-5) = 3 - \frac{1}{6} = \frac{17}{6} \quad .$$

Ans. to 1: $\frac{17}{6}$.

Second Solution of 1:

$$\Delta f = f'(a)\Delta x \quad .$$

Here $a = 5$, $\Delta x = 4 - 5 = -1$. As before $f'(x) = \frac{1}{2\sqrt{4+x}}$, so $f'(5) = \frac{1}{6}$, and

$$\Delta f = \frac{1}{6} \cdot (-1) = -\frac{1}{6} \quad .$$

To get the **linear approximation** we add Δf to $f(a)$, in this problem $f(5) = \sqrt{9} = 3$, so the linear approximation is $f(a) + \Delta f$, which is $3 - \frac{1}{6} = \frac{17}{6}$.

Ans. to 1: $\frac{17}{6}$.

Comments: About half of the students got it correctly. Some people only gave Δf. This is not the **linear approximation**. The linear approximation is $f(a) + \Delta f$. Many people got the sign wrong. They got $\Delta f = 1/6$ instead of $-1/6$. Remember that $\Delta x = 4 - 5 = -1$ is **negative**.

2. Use Newton's method with $x_1 = 2$ to find the second approximation to the equation $x^4 - 17 = 0$.

Solution to 2:

Newton's law says

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad .$$

Here we only need to do it **once**, since the first appx. x_1, is given by the problem. With $n = 1$ we have

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \quad .$$

Here $f(x) = x^4 - 17$, $x_1 = 2$. First we find $f'(x) = 4x^3$, and we have

$$x_2 = 2 - \frac{f(2)}{f'(2)} = 2 - \frac{2^4 - 17}{4 \cdot 2^3} =$$

$$2 - \frac{-1}{4 \cdot 2^3} = 2 - \frac{-1}{32} = \frac{65}{32} \quad .$$

Ans.: $x_2 = \frac{65}{32}$.

Comments: Many people did it one more time. This was not asked for in this problem. If you had to also find x_3, then you would do it again with $n = 2$.