

Solutions to the “QUIZ” for Oct. 23, 2008

1. Find the absolute maximum and the absolute minimum of $f(x) = x^4 - 4x + 1$ for $0 \leq x \leq 2$.

Solution of 1: $f'(x) = 4x^3 - 4$. Setting this to 0, we have to solve

$$4(x^3 - 1) = 0 \quad ,$$

i.e. $x^3 = 1$, that gives $x = 1^{1/3} = 1$. So the only critical number is $x = 1$. Since it happens to be in the interval $[0, 2]$, we keep it as a **finalist**. In addition we take the endpoints $x = 0$ and $x = 2$ to the set of finalists.

So the set of finalists is $\{0, 1, 2\}$.

Now comes the final contest.

$$f(0) = 0^4 - 4 \cdot 0 + 1 = 1 \quad ,$$

$$f(1) = 1^4 - 4 \cdot 1 + 1 = -2 \quad ,$$

$$f(2) = 2^4 - 4 \cdot 2 + 1 = 9 \quad .$$

The **absolute maximum value** is 9 (at $x = 2$), and the **absolute minimum value** is -2 (at $x = 1$).

Comment: About %80 of the people got it right. A few people plugged-in into $f'(x)$ rather than $f(x)$, so be careful!

2. Find the critical numbers of the function $f(x) = 2x^3 - 9x^2 + 12x - 2$.

Solution of 2:

$$f'(x) = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2) = 6(x - 1)(x - 2) \quad .$$

Solving $f'(x) = 0$, we get $6(x - 1)(x - 2) = 0$, whose solutions are $x = 1$ and $x = 2$.

Ans. to 2: The critical numbers are $x = 1$ and $x = 2$.

Comment: About %90 of the people got it right. Some people messed up the algebra.