Solutions to "QUIZ" for Oct. 6, 2008

1. Use rules of differentiation to calculate $f'(x)$ (Do not simplify your answer).

$$f(x) = \sqrt{\sin^{-1} x} \cdot \ln x$$

Solution to 1. First use the **product rule**:

$$f'(x) = ((\sin^{-1} x)^{1/2} \cdot \ln x)' = ((\sin^{-1} x)^{1/2})' \cdot \ln x + (\sin^{-1} x)^{1/2} \cdot (\ln x)'$$

Using the **chain rule** we get that $((\sin^{-1} x)^{1/2})'$ is $(1/2)(\sin^{-1} x)^{-1/2} \cdot (\frac{1}{\sqrt{1-x^2}})$. Also $(\ln x)' = 1/x$. So, combining, we have:

$$f(x)' = (1/2)(\sin^{-1} x)^{-1/2} \cdot (\frac{1}{\sqrt{1-x^2}}) \cdot \ln x + (\sin^{-1} x)^{1/2} \cdot \frac{1}{x} \quad .$$

This is the **answer** (since you were told not to simplify.)

2. Use logarithmic differentiation to find the derivative of the function $f(x) = (2x-1)^3(2x^3+1)^{21}$.

Solution to 2.: We first do the **related** problem of finding $(\ln f(x))'$. First, let's figure-out $\ln f(x)$

$$\ln f(x) = \ln \left[(2x-1)^3(2x^3+1)^{21}\right] = \ln(2x-1)^3 + \ln(2x^3+1)^{21} = 3\ln(2x-1) + 21\ln(2x^3+1) \quad .$$

Now using the famous rule

$$(\ln w)' = \frac{w'}{w} \quad ,$$

and applying it to each piece, we get:

$$(\log f(x))' = 3\frac{(2x-1)'}{2x-1} + 21\frac{(2x^3+1)'}{2x^3+1} = 3\frac{2}{2x-1} + 21\frac{6x^2}{2x^3+1} = \frac{6}{2x-1} + \frac{126x^2}{2x^3+1} \quad .$$

This is the answer to the **related problem**. To get $f'(x)$, we simply multiply the above answer with the input $f(x)$. So:

$$f'(x) = \left(\frac{6}{2x-1} + \frac{126x^2}{2x^3+1}\right)(2x-1)^3(2x^3+1)^{21}.$$

This is the **ans.**.