Solutions to the "QUIZ" of Sept. 11, 2008

1. Assuming that

$$\lim_{x\to 2} f(x) = 1 \quad , \quad \lim_{x\to 2} g(x) = -2 \quad , \quad \lim_{x\to 2} h(x) = 1 \quad ,$$

evaluate the limit

$$\lim_{x\to 2} \; 4f(x) - 2g(x) + 3h(x)$$

Sol. to 1.

$$\lim_{x\to 2} \; 4f(x) - 2g(x) + 3h(x) = \lim_{x\to 2} \; 4f(x) + \lim_{x\to 2} \; -2g(x) + \lim_{x\to 2} \; 3h(x)$$

$$4\left(\lim_{x\to 2} \; f(x)\right) - 2\left(\lim_{x\to 2} \; g(x)\right) + 3\left(\lim_{x\to 2} \; h(x)\right)$$

$$4 \cdot (1) - 2 \cdot (-2) + 3 \cdot (1) = 4 + 4 + 3 = 11.$$

Ans. to 1.: 11.

Comments: %90 of the students got it right, %5 knew how to do it, but messed up (the very simple) arithmetic (remember that $(-2) \cdot (-2) = 4$ **not** -4).

2. For what value of the constant c is the function f continuous for all x? Here

$$f(x) = \begin{cases} cx^3 + 3, & \text{if } x \leq -1; \\ cx^2 - 1, & \text{if } x > -1. \end{cases}$$

Sol. to 2. Since $f(x)$ is obviously continuous before $x = -1$ and after $x = -1$ (since both expressions are continuous), the only possible issue is at $x = -1$. If it is continuous at $x = -1$, then we must first make sure that the limit

$$\lim_{x\to -1} \; f(x)$$

exists. For that, we need to find the limit from the left, and the limit from the right, and set them equal to each other. The limit from the left is:

$$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} (cx^3 + 3) = c(-1)^3 + 3 = -c + 3$$

The limit from the right is:

$$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} (cx^2 - 1) = c(-1)^2 - 1 = c - 1$$

Setting these equal to each other gives

$$-c + 3 = c - 1 \quad ,$$

Solving for c gives

$$4 = 2c \quad ,$$

and we get $c = 2$. So our only hope is $c = 2$. Then the function is:

$$f(x) = \begin{cases} 2x^3 + 3, & \text{if } x \le -1; \\ 2x^2 - 1, & \text{if } x > -1. \end{cases}$$

If you leave it as that, you will probably get full credit, since the question implicitly assumed that there is **one** value of c that makes it. But to be on the safe side, you have to check that $f(-1)$ exists (it does $f(-1) = 1$) and that

$$\lim_{x \to -1} f(x) = f(-1) \quad ,$$

which is true (they are both 1). So our hopes for $c = 2$ materialized, and when $c = 2$ this function is continuous at $x = -1$, and hence everywhere.

Ans. to 2: $c = 2$.

Comments: About %80 of the students got it perfectly, about %10 were on the right track, but messed up the (very simple algebra). Come on people, you should know how to solve for c

$$-c + 3 = c - 1 \quad .$$

The last %10 were completely clueless. These people should come to the free tutoring Monday, Sept. 15, 2008, LSH-143A, where I will go over it in detail.