Solutions to the "QUIZ" for Sept. 22, 2008

1. Compute $f'(2)$, if

$$f(x) = x^2 \quad ,$$

using the limit definition.

Solution to 1: Recall the definition

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \quad .$$

In this problem $a = 2$, $f(x) = x^2$. So

$$f'(2) = \lim_{h \to 0} \frac{(2+h)^2 - 2^2}{h} = \lim_{h \to 0} \frac{2^2 + 2 \cdot 2 \cdot h + h^2 - 4}{h} = \lim_{h \to 0} \frac{4 + 4h + h^2 - 4}{h}$$

$$= \lim_{h \to 0} \frac{4h + h^2}{h} = \lim_{h \to 0} \frac{h(4+h)}{h} = \lim_{h \to 0} 4 + h = 4 + 0 = 4 \quad .$$

Ans.: $f'(2) = 4$.

Comments: About %80 got it perfectly, %10 messed up a little bit, and about %10 are completely clueless.

2. Compute $f'(x)$ from the limit definition (no credit for other methods!), where $f(x) = \frac{1}{x+1}$.

Solution to 2 Recall the definition:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad .$$

Here $f(x) = 1/(x+1)$. We have

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{1+x+h} - \frac{1}{1+x}}{h} =$$

$$\lim_{h \to 0} \frac{\frac{(1+x)-(1+x+h)}{(1+x+h)(1+x)}}{h} = \lim_{h \to 0} \frac{\frac{1+x-1-x-h}{(1+x+h)(1+x)}}{h} =$$

$$\lim_{h \to 0} \frac{\frac{-h}{(1+x+h)(1+x)}}{h}$$

$$= \lim_{h \to 0} \frac{-1}{(1+x+h)(1+x)} = \frac{-1}{(1+x+0)(1+x)} = \frac{-1}{(1+x)(1+x)} = \frac{-1}{(1+x)^2} \quad .$$

Ans. to 2: $f'(x) = \frac{-1}{(1+x)^2}$.

Comments: About %60 got it perfectly, %20 messed up a little bit, %10 started correctly but then lost track, and about %10 are completely clueless.