Solutions to the "QUIZ" for Sept. 25, 2008

1. Differentiate

$$y = \frac{s+2}{s^2-1} \quad .$$

Sol. to 1: By the quotient rule

$$\frac{dy}{ds} = \frac{(s^2-1)(s+2)' - (s+2)(s^2-1)'}{(s^2-1)^2} = \frac{(s^2-1)\cdot 1 - (s+2)(2s)}{(s^2-1)^2} \quad .$$

Using algebra, this equals

$$\frac{s^2-1-2s^2-4s}{(s^2-1)^2} = \frac{-s^2-4s-1}{(s^2-1)^2} = -\frac{s^2+4s+1}{(s^2-1)^2} \quad .$$

Ans. to 1: $\frac{dy}{ds} = -\frac{s^2+4s+1}{(s^2-1)^2}$

2. A particle moves according to a law of motion $s = t^2 - 4t + 8$, $t > 0$, where t is measured in seconds and s is measured in feet.

(a) Find the velocity at any given time t.

(b) What is the velocity after 1 second?

(c) When is the particle at rest?

(d) When is the particle moving in the positive direction?

(e) Find the total distance travelled during the first 3 seconds.

Solution to 2:

(a) $v = \frac{ds}{dt} = 2t - 4$.

(b) $v(1) = 2 \cdot 1 - 4 = -2$. In words: the velocity after one second is 2 feet per second moving in the negative direction.

(c) Setting $v(t) = 0$, we get,

$$2t - 4 = 0 \quad ,$$

solving, we get $t = 2$.

In words: the particle is at rest after two seconds.

(d) It is moving in the positive direction when

$$2t - 4 > 0$$

Solving, gives $t > 2$. In words: it is moving in the positive direction starting at time two seconds, and for ever after.

(e) We have to break the journey into two phases. From $t = 0$ to $t = 2$ it was moving in the negative direction, and since $s(2) - s(0) = (2^2 - 4 \cdot 2 + 8) - 8 = -4$, it traveled $|-4| = 4$ during the negative phase. From $t = 2$ to $t = 3$ it is travelled

$$s(3) - s(2) = (3^2 - 4 \cdot 3 + 8) - (2^2 - 4 \cdot 2 + 8) = 5 - 4 = 1$$

So it travelled 1 foot. Altogether the distance travelled was $4 + 1 = 5$.

Ans. to (e): The total distance travelled during the first three seconds was 5 feet.