Solution to the "QUIZ" for Monday, Sept. 8, 2008

1. : (a) Draw the function

$$f(x) = \begin{cases} x+2, & \text{if } x < 1; \\ 7, & \text{if } x = 1; \\ 2x+2, & \text{if } 1 < x < 2; \\ 5, & \text{if } x = 2; \\ 3x-2, & \text{if } x > 2. \end{cases}$$

Solution. I am too lazy to use a graphing program to plot it, but let me tell you how to do it. Each segment is a straight line. To draw a straight line, all you need are two points on it.

First Piece: the function is $y = x + 2$ for $x < 1$. Pick *any* value of x less than 1, for example $x = 0$, and get the point $(0, 2)$. The other value could be 1 itself (the border), getting the point $(1, 3)$. Now draw the line segment joining the point $(0, 2)$ to the point $(1, 3)$, and extend it **to the left** all the way to $-\infty$. Finally put a **hollow** dot at $(1, 3)$, since the function is $y = x+2$ for $x < 1$ (if it would have been $x \leq 1$, it would have been a filled dot).

Second Piece $y = 2x + 2$ for $1 < x < 2$.

The two points to pick are easy, the left is $x = 1$, so $y = 2 \cdot 1 + 2 = 4$, so the left point is $(1, 4)$, and the right is $x = 2$ so $y = 2 \cdot 2 + 2 = 6$, getting $(2, 6)$. Locate these two points, and draw a line segment from $(1, 4)$ to $(2, 6)$. Put *empty circles* on both of these endpoints, since right now $1 < x < 2$.

Third piece: $y = 3x - 2$ for $x > 2$. When $x = 2$, the corresponding y is $y = 3 \cdot 2 - 2 = 4$, getting the point $(2, 4)$. Now pick any value of x bigger than 2, for example, $x = 3$, and get the point $(3, 7)$. Join $(2, 4)$ and $(3, 7)$, and **extend** this line all the way to $x = \infty$, since $y = 3x - 2$ is valid for all $x > 2$.

Finally, draw **filled-circles** at $(1, 7)$ and $(2, 5)$, corresponding to the function values $f(1)$ and $f(2)$.

(b) From the picture:

$$\lim_{x \to 1^-} f(x) = 3$$

$$\lim_{x \to 1^+} f(x) = 4$$

$$\lim_{x \to 1} f(x) = DNE$$

$$\lim_{x \to 2^-} f(x) = 6$$

$$\lim_{x\to 2^+} f(x) \ = 4$$

$$\lim_{x\to 2} f(x) \ = DNE$$

Comments: **1**. About %60 got it perfectly, another %20 knew how to do it, but made some careless mistakes, and about %20 didn't know how to do it. Those who didn't know how to do it, please come to the free tutoring LSH A-143, this coming Thurs. (9/11), and I will do it in full detail, reviewing the **very important** topic of drawing such *piece-wise* functions.

2. In this problem you had to plot this crazy function, but if you didn't, (or if you are not sure how to plot it, and still want to get the limit questions right), there is a **shortcut**, without referrring to the plot.

To get

$$\lim_{x\to 1^-} f(x)\,,$$

you need to know what is going on immediately to the left of $x = 1$, and we are told that $f(x) = x+2$ for $x < 1$, so to get the **limit from the left**, that is what you would expect the function to be at $x = 1$ by looking to the left of $x = 1$, you simply plug-in $x = 1$ into $y = x + 2$, getting

$$\lim_{x\to 1^-} f(x) = 1 + 2 = 3 \quad .$$

To get

$$\lim_{x\to 1^+} f(x)\,,$$

you need to know what is going immediately to the right of $x = 1$, and we are told that $f(x) = 2x+2$ for $1 < x < 2$, so to get the **limit from the right**, that is what you would expect the function to be at $x = 1$ by looking to the right of $x = 1$, you simply plug-in $x = 1$ into $y = 2x + 2$, getting

$$\lim_{x\to 1^-} f(x) = 2 \cdot 1 + 2 = 4 \quad .$$

To get

$$\lim_{x\to 2^-} f(x)\,,$$

you need to know what is going on immediately to the left of $x = 2$, and we are told that $f(x) = 2x+2$ for $1 < x < 2$, so to get the **limit from the left**, that is what you would expect the function to be at $x = 2$ by looking to the left of $x = 2$, you simply plug-in $x = 2$ into $y = 2x + 2$, getting

$$\lim_{x\to 2^-} f(x) = 2 \cdot 2 + 2 = 6 \quad .$$

To get

$$\lim_{x\to 2^+} f(x)\,,$$

you need to know what is going on immediately to the right of $x = 2$, and we are told that $f(x) = 3x - 2$ for $2 < x$, so to get the **limit from the right**, that is what you would expect the function to be at $x = 2$ by looking to the right of $x = 2$, you simply plug-in $x = 2$ into $y = 3x - 2$, getting

$$\lim_{x\to 2^-} f(x) = 3 \cdot 2 - 2 = 4 \quad .$$