Math 152, Fall 2005, Review Problems for Midterm #1

1. Find the following indefinite integrals.

a) $\displaystyle\int x^2 \cos(3x)\,dx$ b) $\displaystyle\int x \tan^{-1} x\,dx$ c) $\displaystyle\int \sin^5(4x)\,dx$

d) $\displaystyle\int \frac{3x^2+2x+9}{(x+1)(x^2+4)}\,dx$ e) $\displaystyle\int \frac{dx}{x\sqrt{1-4x^2}}$ f) $\displaystyle\int \sec^4(5x)\tan(5x)\,dx$

2. The region R is bounded by the curves $y = 2x$, $y = \sqrt{1+3x^2}$ and the y-axis. Find the volume of the solid obtained by rotating R: (a) about the x-axis; (b) about the y-axis.

3. The base of a solid is the region inside the curve $x^4 + y^2 = 1$. Each cross-section of the solid perpendicular to the x-axis is a square. What is the volume of the solid?

4. Consider the curve $y = \sin x$, $0 \le x \le \pi$. Set up integrals representing (a) the length of this curve; (b) the area of the surface formed when the curve is revolved about the x-axis. Evaluate one of these integrals. (The other one can be approximated by numerical methods, but you're not asked to do so.)

5. A person using a spring-like exercise machine is able to stretch the spring a distance of 2 feet from rest by expending W_1 units of work. If the spring had been stretched a distance of 3 feet instead, then the work required would have been W_2 units. Find the ratio W_2/W_1.

6. Determine whether each of the following integrals is convergent or divergent. Evaluate those that are convergent. Be sure to show your work and explain your reasoning.

a) $\displaystyle\int_5^\infty \frac{dx}{x\,(\ln x)}$ b) $\displaystyle\int_2^\infty \frac{x^{14}+1}{x^{15}-x}\,dx$ c) $\displaystyle\int_1^4 \frac{dx}{\sqrt{4-x}}$ d) $\displaystyle\int_0^1 \frac{\ln x}{\sqrt{x}}\,dx$

7. Find the average value A of $\tan^2 x$ on the interval $0 \le x \le \pi/4$. Is A larger or smaller than the average of the max and min values of $\tan^2 x$ on the interval? Draw a picture which explains this.

8. Let $I = \displaystyle\int_0^2 e^{x^2}\,dx$.

a) Using the Trapezoidal Rule with $n = 4$ subdivisions gives what approximate value for this integral? (Give an exact answer in terms of e.)

b) Let T_n be the approximation to I obtained from the Trapezoidal Rule with n subdivisions. Find a number M such that

$$|I - T_n| < 10^{-8} \quad \text{whenever} \quad n > M.$$

The number M can be expressed in terms of e. There is no need to use a calculator.

9. Find the equation for the curve in the half-plane $x \ge 0$ that passes through the point $(x, y) = (0, 3)$ and whose slope at any point (x, y) is $y - y^2$. Plot this curve and find any asymptotes.