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**Formula sheet for Math 152, Exam 2**

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$$\lim_{n \rightarrow \infty} n^{1/n} = 1; \quad \lim_{n \rightarrow \infty} \frac{x^n}{n!} = 0; \quad \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e; \quad \lim_{n \rightarrow \infty} \frac{n^k}{a^n} = 0 \text{ if } a > 1.$$

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$$\lim_{n \rightarrow \infty} c^n = 0 \text{ if } |c| < 1; \quad \sum_{n=0}^{\infty} c^n = \frac{1}{1-c} \text{ if } |c| < 1.$$

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$$\sum_{n=1}^{\infty} \frac{1}{n^p} \text{ converges if } p > 1 \text{ (and diverges if } p \leq 1).$$

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If the statement  $\lim_{n \rightarrow \infty} a_n = 0$  is false, then  $\sum_{n=1}^{\infty} a_n$  diverges.

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If  $0 \leq a_n \leq b_n$  and  $\sum_{n=1}^{\infty} b_n$  converges, then  $\sum_{n=1}^{\infty} a_n$  converges.

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If  $0 \leq a_n \leq b_n$  and  $\sum_{n=1}^{\infty} a_n$  diverges, then  $\sum_{n=1}^{\infty} b_n$  diverges.

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If  $0 < a_n, 0 < b_n, 0 < \lim_{n \rightarrow \infty} \frac{a_n}{b_n} < \infty$ , then  $\sum_{n=1}^{\infty} a_n$  and  $\sum_{n=1}^{\infty} b_n$  both converge or both diverge.

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If  $f(x)$  is a positive decreasing continuous function and  $a_n = f(n)$  then  
$$\int_{n+1}^{\infty} f(x) dx \leq a_{n+1} + a_{n+2} + a_{n+3} + \cdots \leq \int_n^{\infty} f(x) dx.$$

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If  $a_n > 0, a_1 \geq a_2 \geq a_3 \geq \cdots$  and  $\lim_{n \rightarrow \infty} a_n = 0$  then  $\sum_{n=1}^{\infty} (-1)^n a_n$  converges.

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$\sum a_n$  converges absolutely when  $\sum |a_n|$  converges.  $\sum a_n$  converges conditionally when it converges, but does not converge absolutely. If  $\sum a_n$  converges absolutely, then  $\sum a_n$  converges.

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If  $a_n > 0$  and  $\lim_{n \rightarrow \infty} \frac{|a_{n+1}|}{|a_n|} = L$  then  $\left\{ \begin{array}{l} \sum a_n \text{ converges absolutely if } L < 1, \\ \sum a_n \text{ diverges if } L > 1, \\ \text{the test is inconclusive if } L = 1. \end{array} \right.$

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If  $a_n \geq 0$  and  $\lim_{n \rightarrow \infty} |a_n|^{1/n} = L$  then  $\left\{ \begin{array}{l} \sum a_n \text{ converges absolutely if } L < 1, \\ \sum a_n \text{ diverges if } L > 1, \\ \text{the test is inconclusive if } L = 1. \end{array} \right.$

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The  $n$ th Taylor polynomial of  $f(x)$  with center  $a$  is  $T_n(x) = \sum_{k=0}^n \frac{f^{(k)}(a)}{k!} (x-a)^k$ . The  $n$ th remainder term is  $R_n(x) = f(x) - T_n(x)$ .

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If  $|f^{(n+1)}(x)| \leq M$  for  $|x-a| \leq d$ , then  $|R_n(x)| \leq \frac{M}{(n+1)!} |x-a|^{n+1}$  for  $|x-a| \leq d$ .

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The Taylor series of  $f(x)$  with center  $a$  is  $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$ .

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$$e^{ix} = \cos x + i \sin x; \quad (1+x)^k = 1 + \sum_{n=1}^{\infty} \frac{k(k-1)(k-2)\cdots(k-n+1)}{n!} x^n \text{ if } |x| < 1.$$

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