Dr. Z's Math152 Handout #10.2 [Calculus with Parametric Curves]

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Problem Type 10.2a: Find an equaion of the tangent line to the curve at the point corresponding to the given vale of the parameter

$$x = f(t) \quad , \quad y = g(t) \quad ; \quad t = a \quad ,$$

where $f(t)$, $g(t)$ are expressions in t and a is some number.

Example Problem 10.2a: Find an equaion of the tangent line to the curve at the point corresonding to the given vale of the parameter

$$x = 2t^2 + 1, \quad y = \frac{1}{3}t^3 - t \quad ; \quad t = 3 \quad .$$

Steps	**Example**
1. Compute $\frac{dy}{dt}$ and $\frac{dx}{dt}$, getting expressions in t. Then divide the former by the latter to get $\frac{dy}{dx}$, a certain exprssion in t. You use $$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$	**1.** $\frac{dy}{dt} = y'(t) = t^2 - 1$, $\frac{dx}{dt} = x'(t) = 4t$,hence $\frac{dy}{dx} = (t^2 - 1)/4t$.
2. Plug-in $t = a$ to get the **slope** at the point. Also plug-in $t = a$ into x and y to get the **point**.	**2.** When $t = 3$, the slope is $\frac{dy}{dx} = (t^2 - 1)/4t\|_{t=3}$, that equals $(3^2 - 1)/(4 \cdot 3) = \frac{2}{3}$. So $m = \frac{2}{3}$. To get the point we have $x(3) = 2 \cdot (3)^2 + 1 = 19$ and $y(3) = (1/3) \cdot 3^3 - 3 = 6$. So the **point** is $(19, 6)$.
3. Use the famous point-slope equation from geometry to find the equation of the line: $(y - y_0) = m(x - x_0)$.	**3.** $y - 6 = (2/3)(x - 19)$, that simplifies to $y = \frac{2}{3}x - 20/3$ **Ans.**: The equation of the tangent line is $y = \frac{2}{3}x - \frac{20}{3}$.

Problem Type 10.2b: Set-up, but do not evaluate, an integral that represents the length of the curve

$$x = f(t) \quad , \quad y = g(t) \quad ; \quad a \leq t \leq b \quad ,$$

where $f(t)$, $g(t)$ are expressions in t and a and b are some numbers ($a < b$).

Example Problem 10.2b: Set-up, but do not evaluate, an integral that represents the length of the curve

$$x = 1 + e^t, \quad y = t^2 \quad ; \quad -3 \le t \le 3 \quad .$$

Steps	**Example**
1. Compute $\frac{dy}{dt}$ and $\frac{dx}{dt}$, getting expressions in t.	**1.** $\frac{dx}{dt} = e^t$, $\frac{dy}{dt} = 2t$.
2. Use the formula for the arclength $$\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$	**2.** The arclentgh is $$\int_{-3}^{3} \sqrt{(e^t)^2 + (2t)^2}\, dt = \int_{-3}^{3} \sqrt{e^{2t} + 4t^2}\, dt \quad .$$ This is the **answer**!. You were not asked to evaluate the integral.