Dr. Z's Math152 Handout #11.1 [Sequences]

By Doron Zeilberger

Problem Type 11.1a: Determine whether the sequence

$$a_n = f(n)$$

(where $f(x)$ is a 'nice' function) converges or diverges. If it converges find its limit.

Example Problem 11.1a: Determine whether the sequence

$$a_n = \frac{3+5n^2}{n+n^2} \quad ,$$

converges or diverges. If it converges find its limit.

Steps	**Example**
1. Replace the n by x and use your Calc I expertise (you can consult: http://www.math.rutgers.edu/~zeilberg /sod/ho2.pdf) to compute $\lim_{x\to\infty} f(x)$.	**1.** To compute $\lim_{x\to\infty} \frac{3+5x^2}{x+x^2}$. note that when x gets bigger and bigger, 3 is insignificant next to $5x^2$ and x is insignificant next to x^2 (what I call 'forget about the little ones'), so the limit can be replaced by the simpler limit $\lim_{x\to\infty} \frac{5x^2}{x^2} = 5$.
2. If that limit diverges, then the sequence diverges. If it converges then the sequence converges and the value of the limit is the same.	**2.** The above limit exists and is equal to 5, so **Ans.:** The sequence converges and its limit equals 5.

Problem Type 11.1b: Determine whether the sequence

$$a_n = (-1)^n f(n)$$

(where $f(x)$ is a 'nice' function) converges or diverges. If it converges find its limit.

Example Problem 11.1b: Determine whether the sequence

$$a_n = \frac{(-1)^n(3+5n^2)}{n+n^2} \quad ,$$

converges or diverges. If it converges find its limit.

Steps | **Example**

1. Replace the n by x and use your Calc I expertise (you can consult:

http://www.math.rutgers.edu/ zeilberg

/sod/ho2.pdf) to compute

$$\lim_{x\to\infty} f(x) \quad .$$

1. To compute

$$\lim_{x\to\infty} \frac{3+5x^2}{x+x^2} \quad ,$$

note that when x gets bigger and bigger, 3 is insignificant next to $5x^2$ and x is insignificant next to x^2 (what I call 'forget about the little ones'), so the limit can be replaced by the simpler limit

$$\lim_{x\to\infty} \frac{5x^2}{x^2} = 5 \quad .$$

2. If that limit diverges, then the sequence definitely diverges. But even it converges to something other than 0, the sequence, because of the $(-1)^n$ in it, still diverges. Only if the above limit is 0 does the sequence converge, and then the limit of the sequence is 0.

2. While the above limit exists, it is not equal to 0, so

Ans.: The sequence diverges, because in the long-run it is like $5, -5, 5, -5, \ldots$ so it can't make up its mind.

Problem Type 11.1b': Determine whether the sequence

$$a_n = (-1)^n f(n)$$

(where $f(x)$ is a 'nice' function) converges or diverges. If it converges find its limit.

Example Problem 11.1b': Determine whether the sequence

$$a_n = \frac{(-1)^n(3+5n^2)}{n+n^3} \quad ,$$

converges or diverges. If it converges find its limit.

Steps | **Example**

1. Replace the n by x and use your Calc I expertise (you can consult:

http://www.math.rutgers.edu/ zeilberg

/sod/ho2.pdf) to compute

$$\lim_{x \to \infty} f(x) \quad .$$

1. To compute

$$\lim_{x \to \infty} \frac{3+5x^2}{x+x^3} \quad ,$$

note that when x gets bigger and bigger, 3 is insignificant next to $5x^2$ and x is insignificant next to x^3 (what I call 'forget about the little ones'), so the limit can be replaced by the simpler limit

$$\lim_{x \to \infty} \frac{5x^2}{x^3} = \lim_{x \to \infty} \frac{5}{x} = 0 \quad .$$

2. If that limit diverges, then the sequence definitely diverges. But even it converges to something other than 0, the sequence, because of the $(-1)^n$ in it, still diverges. Only if the above limit is 0 does the sequence converge, and then the limit of the sequence is 0.

2. Not only does the above limit exist, it is equal to 0! so

Ans.: The sequence converges to 0.