

Dr. Z's Math152 Handout #7.7 [Approximate Integration]

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**Problem Type 7.7a:** Use (a) The Trapezoid Rule, (b) The Midpoint Rule, and (c) Simpson's Rule to approximate the given integral with the specified value of  $n$

$$\int_a^b f(x) dx \quad , \quad n = \text{EvenInteger}$$

**Example Problem:** Use (a) The Trapezoid Rule, (b) The Midpoint Rule, and (c) Simpson's Rule to approximate the given integral with the specified value of  $n$

$$\int_2^6 \frac{1}{x} dx \quad , \quad n = 8$$

**Steps**

1. Find  $\Delta x = (b - a)/n$ , and write down

$$x_0 = a, x_1 = a + \Delta x, x_2 = a + 2\Delta x, \dots,$$

$$x_n = a + n\Delta x = b \dots,$$

For the Midpoint rule, you also need the midpoints of  $[x_i, x_{i+1}]$  (call it  $\bar{x}_i$ ) (for  $i = 1, \dots, n$ ).

**Example**

1.  $\Delta x = (6 - 2)/8 = .5$ .

$$x_0 = 2, x_1 = 2.5, x_2 = 3, x_3 = 3.5, x_4 = 4,$$

$$x_5 = 4.5, x_6 = 5, x_7 = 5.5, x_8 = 6$$

$$\bar{x}_1 = 2.25, \bar{x}_2 = 2.75, \bar{x}_3 = 3.25, \bar{x}_4 = 3.75,$$

$$\bar{x}_5 = 4.25, \bar{x}_6 = 4.75, \bar{x}_7 = 5.25, \bar{x}_8 = 5.75,$$

**2.** For the Midpoint Rule the approximation, called  $M_n$  is

$$M_n = \Delta x [f(\bar{x}_1) + f(\bar{x}_2) + \dots + f(\bar{x}_n)]$$

For the Trapezoid Rule the approximation, called  $T_n$  is

$$T_n = \frac{\Delta x}{2} [f(x_0) +$$

$$2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)]$$

For Simpson's Rule the approximation, called  $M_n$  is (now  $n$  must be even!)

$$S_n = \frac{\Delta x}{3} [f(x_0) +$$

$$4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

**2.**

$$M_8 = (.5)[f(2.25) + f(2.75) + f(3.25) + f(3.75) +$$

$$f(4.25) + f(4.75) + f(5.25) + f(5.75)] =$$

$$(.5)[1/(2.25) + 1/(2.75) + 1/(3.25) + 1/(3.75) +$$

$$1/(4.25) + 1/(4.75) + 1/(5.25) + 1/(5.75)] = 1.0936247\dots$$

$$T_8 = \frac{(.5)}{2} [f(2) + 2 \cdot (f(2.5) + f(3) + f(3.5) + f(4)$$

$$+ f(4.5) + f(5) + f(5.5)) + f(6)]$$

$$(.25)[1/2 + 2 \cdot (1/(2.5) + 1/3 + 1/3.5 + 1/4$$

$$+ 1/(4.5) + 1/5 + 1/5.5) + 1/6] = 1.10321067\dots$$

$$S_8 = \frac{(.5)}{3} [f(2) + 4f(2.5) + 2f(3) + 4f(3.5) + 2f(4) +$$

$$4f(4.5) + 2f(5) + 4f(5.5) + f(6)]$$

$$= (1/6)[1/2 + 4/(2.5) + 2/3 + 4/3.5 + 2/4 + 4/4.5 +$$

$$2/5 + 4/(5.5) + 1/6] = 1.09872534\dots$$