NAME: (print!) ____

Section: ____ **E-Mail address:** ____________________

MATH 251 (1-3), Dr. Z. , FINAL, 8:00-11:00am , Friday, Dec. 15, 2006 [Blue Version]

FRAME YOUR FINAL ANSWER(S) TO EACH PROBLEM
Do not write below this line

---

1. (out of 13)
2. (out of 13)
3. (out of 13)
4. (out of 13)
5. (out of 13)
6. (out of 13)
7. (out of 13)
8. (out of 13)
9. (out of 12)
10. (out of 12)
11. (out of 12)
12. (out of 12)
13. (out of 12)
14. (out of 12)
15. (out of 12)
16. (out of 12)

1. (13 pts.) Find the curvature of the curve

$$\mathbf{r}(t) = \langle t, t^2, \frac{2}{3}t^3 \rangle$$

at the point $(1, 1, \frac{2}{3})$.

2. (13 points) By using Stokes's Theorem, or otherwise, evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where

$$F(x, y, z) = yz^2\,\mathbf{i} + xz^2\,\mathbf{j} + 2xyz\,\mathbf{k} \quad ,$$

and C is the curve of intersection of the plane $x + y + z = 1$ and the cylinder $x^2 + y^2 = 9$ oriented counterclockwise as viewed from above. Be sure to explain everything.

3. (13 points) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if

$$x^3 + y^3 + z^3 = 5xyz + 1 \quad .$$

4. (13 points) Find an equation for the tangent plane to the parametric surface:

$$x = u^2 \quad , \quad y = u + v \quad , \quad z = v^2 \quad ,$$

at the point $(1, 2, 1)$. Simplify as much as you can!

5. (13 points) Change the order of integration in

$$\int_1^2 \int_0^{\ln x} f(x,y)\,dy\,dx \quad .$$

6. (13 points) Let

$$\mathbf{F}(x,y,z) =$$

$$\langle \cos(\sqrt{1+x}+x^3)\,,\ \tan((1+\cos(\sqrt{1+x}+x^3))^7)\,,\ \tan^{-1}((e^{x^2}+\cos(\sqrt{1+x}+x^3))^7\rangle \quad ,$$

and let $\langle\, P\,,\, Q\,,\, R\,\rangle = curl\,\mathbf{F}$. Compute

$$\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z} \quad .$$

Be sure to explain everything.

7. (13 points) Let C be the line segment from $(0, 1)$ to $(3, 5)$, find $\int_C 2xy\, ds$.

8. (13 points) Evaluate

$$\int_C (5y - \sin(e^x))\, dx + (10x - e^{\cos^2 y})\, dy \quad ,$$

where C is the closed curve consisting of the boundary of the rectangle

$$\{\, (x, y) \,|\, 0 \le x \le 4\,,\ 0 \le y \le 3 \,\} \quad .$$

9. (12 points) Find the Jacobian of the transformation

$$x = u + v + w \quad , \quad y = u^2 + v^2 + w^2 \quad , \quad z = u^3 + v^3 + w^3 \quad .$$

Simplify as much as you can!

10. (12 points) Find the volume of the solid bounded by the cylinder $y = x^2$ and planes $z = 0$ and $y + z = 1$. Simplify as much as you can!

11. (12 points) Use Lagrange multipliers (no credit for other methods!) to find the largest value that $x + 3y + 5z$ can be, given that $x^2 + y^2 + z^2 = 35$.

12. (12 points) Find an equation of the tangent plane to the surface $z = e^{2x-3y}$ at the point $(3, 2, 1)$. Simplify as much as you can!

13. (12 points) Find the local maximum and minimum points, the local maximum and minimum values, and saddle point(s) of the function $f(x, y) = 4x^2 + y^2 + 2x^2y - 1$.

14. (12 points) Find the velocity and position vectors of a particle whose acceleration is $\mathbf{a}(t) = \mathbf{i} + \mathbf{j}$, and at $t = 0$ the velocity is $\mathbf{i} - \mathbf{j}$ and the position is $\mathbf{k}$.

15. (12 points) Find an equation for the plane through the point $(1, 0, 2)$ that contains the line

$$\mathbf{r}(t) = \langle 1, 1, 1\rangle + t\langle 1, -1, 0\rangle \quad .$$

Simplify as much as you can!

16. (12 points) Compute the limit

$$\lim_{(x,y,z)\to(1,1,1)} \; e^{-xy}\sin(\pi z/2) \quad ,$$

or prove that it does not exists.