NAME: (print!) ________________________

Section: ___ **E-Mail address:** ________________

MATH 251 (4-6), Dr. Z. , FINAL, noon-3:00pm , Thurs., Dec. 21, 2006 [Blue Version]

FRAME YOUR FINAL ANSWER(S) TO EACH PROBLEM
Do not write below this line

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1. (out of 13)

2. (out of 13)

3. (out of 13)

4. (out of 13)

5. (out of 13)

6. (out of 13)

7. (out of 13)

8. (out of 13)

9. (out of 12)

10. (out of 12)

11. (out of 12)

12. (out of 12)

13. (out of 12)

14. (out of 12)

15. (out of 12)

16. (out of 12)

1. (13 points) Evaluate

$$\int_0^4 \int_{y/4}^1 \frac{12}{(x^2+1)^4}\, dx\, dy \quad ,$$

by inverting the order of integration and evaluating the new iterated integral.

2. Suppose that the **position** of a certain particle is given by

$$\mathbf{r}(t) = \langle e^t \cos t\,,\ e^t \sin t\,,\ e^t\,\rangle \quad ,0 \le t \le \pi \quad .$$

(a) (4 points) Find the **velocity** of the particle as a function of the time t.

(b) (9 points) Find the **length of the arc** traversed by the moving particle for $0 \le t \le \pi$.

3. (13 points) Find $\frac{\partial f}{\partial r}$ and $\frac{\partial f}{\partial s}$ as **functions of** r **and** s , if

$$f(x, y) = x^3 + 2xy + y^3 \quad ,$$

and the variables are related by $x = r - s$ and $y = r + s$. You do not need to simplify!

4. Let

$$f(x, y, z) = -x^2 + y^2 + z^2 - 1 \quad .$$

(a) (2 points) Compute $\bigtriangledown f$.

(b) (5 points) Find a normal to the level surface $f(x, y, z) = 0$ at the point $(1, 1, 1)$, and give an equation for the tangent plane to that surface at that point.

(c) (6 points) Compute the directional derivative of $f(x, y, z)$ at the point $(1, 1, 1)$ in the direction $\langle 1\,,\, 2\,,\, 2 \rangle$.

5. (13 points) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if

$$\sin(x + 2y + 3z) = 5xyz + 1 \quad .$$

6. (13 points) Use polar coordinates to compute the double integral

$$\int\int_D xy\, dA \quad ,$$

where

$$D = \{(x,y)\,|\, x^2 + y^2 \leq 4\,,\ x \geq 0\,,\ y \geq 0\,\} \quad .$$

(Hint: recall the trig identity $\sin 2\theta = 2 \sin\theta \cos\theta$)

7. (13 points) Use the transformation

$$x = 2u + v \quad , \quad y = u + 2v \quad ,$$

to evaluate the integral

$$\int\int_R (2x - y)\, dA$$

where R is the triangular region with vertices $(0, 0)$, $(2, 1)$, and $(1, 2)$.

8. (13 points) By using Stokes' theorem, or otherwise, evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r} \quad ,$$

where $\mathbf{F}$ is the vector field

$$\mathbf{F}(x, y, z) = \langle\, 2xy^2z^2 \,,\, 2x^2yz^2 \,,\, 2x^2y^2z \,\rangle \quad ,$$

and C is the **closed** curve going from $(1, 0, 1)$ to $(3, 4, 9)$, and then from $(3, 4, 9)$ to $(-1, 4, 11)$, and then from $(-1, 4, 11)$ to $(5, 2, 11)$ and finally from $(5, 2, 11)$ back to the starting point $(1, 0, 1)$. Explain everything!

9.

(a) (4 points) Compute the surface integral

$$\int\int_S 8\, dS \quad ,$$

where S is the sphere $(x-1)^2 + (y+4)^2 + (z-9)^2 = 100$.

(b) (4 points) Compute the triple integral

$$\int\int\int_E 30\, dV \quad ,$$

where E is the ball $\{\, (x, y, z) \,|\, (x-1)^2 + (y+4)^2 + (z-9)^2 \leq 100 \,\}$.

(c) (4 points) Compute the line integral

$$\int_C 3\, ds \quad ,$$

where C is the circumference of the region $\{(x, y) \,|\, x^2 + y^2 \leq 4\,,\ y \geq 0\}$.

10. (12 points) Evaluate the iterated integral by converting it to polar coordinates

$$\int_0^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2}\, dy\, dx \quad .$$

11. (12 points) Evaluate the surface integral

$$\int\int_S \sqrt{3}\, x\, dS \quad ,$$

where S is the triangular region with vertices $(1,0,0), (0,1,0), (0,0,1)$.

12. (12 points) Determine whether or not the vector field

$$F(x, y, z) = (e^x + yz)\,\mathbf{i} + (e^y + xz)\,\mathbf{j} + (e^z + xy)\,\mathbf{k}$$

is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

13. (12 points) By using Green's theorem, or otherwise, evaluate the line integral

$$\int_C e^y \, dx + 2xe^y \, dy \quad ,$$

where C goes from $(0,0)$ to $(1,0)$, then from $(1,0)$ to $(1,1)$, then from $(1,1)$ to $(0,1)$, and then from $(0,1)$ back to $(0,0)$.

14. (12 points) Show that the line integral

$$\int_C 2x \sin y \, dx \; + \; (x^2 \cos y - 3y^2) \, dy \quad ,$$

is independent of the path C, and evaluate it if C is *any* path from $(1, 0)$ to $(0, 2)$.

15. (12 points) Evaluate

$$\iiint_B 5\,(x^2 + y^2 + z^2)^2\, dV$$

where B is the ball

$$\{\,(x, y, z)\,|\, x^2 + y^2 + z^2 \le 4\,\} \quad .$$

16. (12 points) Find the local maximum and minimum point(s), the local maximum and minimum values, and saddle point(s) of the function

$$f(x,y) = 6y^2 - 2y^3 + 3x^2 + 6xy \quad .$$