Dr. Z.’s Calc4 Homework assignment 9

Version of Oct. 7, 2013: [correcting typos in 1(b), thanks to Christopher Gurd]. Version of Oct. 6, 2013: [correcting minor typos in 1(d) and 1(e), thanks to Christopher Gurd]. Previous Versions: Oct. 5, 2013: [correcting minor typos, that do no change the answers (previously \( y'(t) \) was \( y(t) \)].

1. For each of the following diff.eq. initial value problems, and intervals, decide whether the theorem promises you that there is a unique solution. Explain!

   a. \((t^3 - 1) y''(t) + \sin t y'(t) + (t^3 + 1) y(t) = e^t\) , \(y(0) = 1\) , \(y'(0) = 3\) ; \(-2 < t < 2\)

   b. \((t^3 + 1) y''(t) + \sin t y'(t) + (t^3 + 1) y(t) = e^t\) , \(y(\frac{3}{2}) = 1\) , \(y'(\frac{3}{2}) = 3\) ; \(1 < t < 2\)

   c. \((t^3 - 8) y''(t) + \sin t y'(t) + (t^3 + 1) y(t) = e^t\) , \(y(0) = 1\) , \(y'(0) = 3\) ; \(-3 < t < 1\)

   d. \(t^2 y''(t) + \sin t y'(t) + (t^3 + 1) y(t) = e^t\) , \(y(\frac{1}{2}) = 1\) , \(y'(\frac{1}{2}) = 3\) ; \(\frac{1}{1000} < t < 1\)

   e. \(y''(t) + \csc t y'(t) + (t^3 + 1) y(t) = e^{t^2}\) , \(y(0) = 1\) , \(y'(0) = 3\) ; \(-1 < t < 1\)

2. Find the largest open interval for which the solutions of the following diff.eqs. exist and are unique. Explain!

   a. \((t - 1)(t - 4) y''(t) + \sin t y'(t) + \cos t y(t) = t\) , \(y(0) = 1\) , \(y'(0) = 3\)

   b. \((t + 100)(t - 200) y''(t) + \sin t y'(t) + \cos(e^t)y(t) = t^3\) , \(y(10) = 1\) , \(y'(10) = 3\)

   c. \((t^2 + 1) y''(t) + \sin t y'(t) + \cos(e^t)y(t) = e^{-t^2}\) , \(y(3) = -1\) , \(y'(3) = 13\)

   d. \((t - 3)(t - 1)(t + 1)(t + 5) y''(t) + \sin t y'(t) + \cos(e^t)y(t) = t^3\) , \(y(0) = 1\) , \(y'(0) = 3\)

3. Find the Wronskian, \(W(f(t), g(t))\) of the following pair of functions.

   a. \(f(t) = e^{t^2}\) , \(g(t) = te^{t^2}\)

   b. \(f(t) = e^{3t}\) , \(g(t) = 3e^{3t}\)

   c. \(f(t) = e^{2t}\) , \(g(t) = te^{2t}\)

   d. \(f(t) = te^{2t}\) , \(g(t) = te^{3t}\)

4. Find the Wronskian (up to a constant in front) of any two solutions of the following diff.eq.

   a. \(y''(t) + e^{-t} y'(t) + e^t y(t) = 0\) .
b. 
\[ ty''(t) + t \sin 3t \ y'(t) + e^t \ y(t) = 0 \].

c. 
\[ y''(t) + 3y'(t) + 2y(t) = 0 \].