All Dr. Z.’s Calc4 Homework assignments  

Dr. Z.’s Calc4 Homework assignment 1  

1. Draw a direction field for the given differential equations. Based on the direction field, determine the behavior of $y$ as $t \to \infty$

   a. $y' = 20 - 5y$
   
   b. $y' = 10 + 2y$
   
   c. $y' = (y + 1)(y - 2)$
   
   d. $y' = (y - 2)(y - 4)(y - 6)$

2. Verify that $y(t) = e^t + 2e^{-t}$ is a solution of the initial value differential equation

   $$y'' - y = 0, \quad y(0) = 3, \quad y'(0) = -1.$$  

3. Verify that $y(t) = \tan t$ is a solution of the initial value differential equation

   $$y'(t) = 1 + y(t)^2, \quad y(0) = 0.$$  

4. For each of the following diff.eq. state whether there are linear or non-linear, and find the order.

   a. $y'''(t) + 6ty'(t) + (\cos t) y(t) = 7$
   
   b. $y''''(t) + 6y'(t) + y(t) = 7$
   
   c. $y''(t)^3 + y'(t) + 3y(t) = t$
   
   d. $y^{(100)}(t) + y'(t)y(t) = 6$

Dr. Z.’s Calc4 Homework assignment 2

Find a solution of the given initial value problems.

1. $y' + 3y = te^{3t}, \quad y(0) = 1.$

2. $y' - y = e^t, \quad y(1) = 0.$

3. $y' - 2y = e^{2t}, \quad y(0) = 1.$

4. $y' + (2/t)y = (\cos t)/t^2, \quad y(\pi) = 0, \quad t > 0.$

5. $ty' + 3y = \cos t, \quad y(\pi) = 0, \quad t > 0.$
6. \( ty' + (t + 1)y = 2t \), \( y(1) = 0 \).

Find the General solutions:

7. \( ty' + (t + 1)y = 3te^{-t} \)

8. \( 2y' - y = e^{2t} \)

Dr. Z.’s Calc4 Homework assignment 3

Solve the differential equation

1. Solve the differential equation
   \[ y' = \frac{x^3}{y^2} \]

2. Solve the differential equation
   \[ y' = \frac{x^3}{y(1 + x^4)} \]

3. Solve the differential equation
   \[ y' + y^3 \cos 2x = 0 \]

4. Solve the differential equation
   \[ xy' = \sqrt{9 - y^2} \]

5. Solve the following initial value problem
   \[ y' = x^2y^3 \quad y(1) = 2 \]

6. Solve the following initial value problem
   \[ y' = \sin x \sec y \quad y\left(\frac{\pi}{2}\right) = \frac{\pi}{4} \]

7. Find an equation of the curve that passes through the point \((1, 1)\) and whose slope at \((x, y)\) is \(y^2/x^2\).

8. Find an equation of the curve that passes through the point \((1, 2)\) and whose slope at \((x, y)\) is \(y^2/x^5\).

Dr. Z.’s Calc4 Homework assignment 4

1. Without actually solving the diff.eq.s decide whether the following initial value problems are guaranteed to have unique solutions, on the specified intervals. **Explain!**
a. \( y'(t) - (t^4 + 1) y(t) = \sin^3 t, \quad -100 < t < 10 \), \( y(1) = 3 \)

b. \( y'(t) + \frac{1}{t^2} y(t) = \sin^3 t, \quad 0 < t < 2 \), \( y(1) = 10 \)

c. \( y'(t) + \frac{1}{(t+3)^2} y(t) = \sqrt{t}, \quad 3 < t < 5 \), \( y(4) = -100 \)

d. \( y'(t) + \frac{1}{(t+3)^2} y(t) = e^{-t^4}, \quad -10 < t < 1 \), \( y(-4) = 0 \)

e. \( y'(t) + \cos^2 t \ y(t) = \frac{1}{t+3}, \quad -\pi/2 < t < \pi/2 \), \( y(0) = 1 \)

f. \( y'(t) + \sin t \ y(t) = \frac{1}{t+3}, \quad -\pi < t < \pi \), \( y(0) = 1 \)

2. Find the maximal open intervals for which the following first-order diff. eq. is guaranteed to have a unique solution. Explain!

\[
y'(t) + \frac{t^2}{(t+1)^2(t-1)^3(t-2)} y(t) = \frac{t^5}{t-4}. 
\]

3. Find the maximal open intervals for which the following first-order diff. eq. is guaranteed to have a unique solution. Explain!

\[
y'(t) + \frac{t^2}{1+t^4} y(t) = \sqrt{|t|}. 
\]

4. For which of the following initial value problems are we guaranteed to have a unique solution, \( y(t) \) on some interval of the \( t \)-line, around the initial time? Explain!

a. \( y' = \frac{1}{1+2y-3t}, \quad y(1) = 3 \)

b. \( y' = \frac{1}{10-2y-4t}, \quad y(1) = 3 \)

c. \( y' = \frac{1}{1+y^2+t^3}, \quad y(0) = 1 \)

d. \( y' = t^3 + y^3, \quad y(0) = 0 \)

Dr. Z.’s Calc4 Homework assignment 5

1. For the following diff. eq. determine the critical (equilibrium) solution and decide whether it is asymptotically stable, unstable, or semi-stable.

\[
\frac{dy}{dt} = 2y - 6. 
\]

2. For the following diff. eq. determine the critical (equilibrium) solution and decide whether it is asymptotically stable, unstable, or semi-stable.

\[
\frac{dy}{dt} = 10 - 5y. 
\]
3. For the following diff. eq. determine the critical (equilibrium) solution(s) and decide whether they are asymptotically stable, unstable, or semi-stable.

\[ \frac{dy}{dt} = y^2 - 4 \]

4. For the following diff. eq. determine the critical (equilibrium) solution(s) and decide whether it is asymptotically stable, unstable, or semi-stable.

\[ \frac{dy}{dt} = 3(y - 3)^4 \]

5. For the following diff. eq. determine the critical (equilibrium) solution(s) and decide whether they are asymptotically stable, unstable, or semi-stable.

\[ \frac{dy}{dt} = y(y^2 - 4) \]

6. For the following diff. eq. determine the critical (equilibrium) solution(s) and decide whether they are asymptotically stable, unstable, or semi-stable.

\[ \frac{dy}{dt} = y^2(y - 1) \]

**Dr. Z.’s Calc4 Homework assignment 6**

1. For the following first-order diff.eqs., decide whether or not they are exact. If they are, solve them.

   a. \[ (2xy^2 + y^3 + y) + (2x^2y + 3xy^2 + 4x) y' = 0 \]

   b. \[ (e^{x+y} + 2) + (e^{x+y} + 3) y' = 0 \]

   c. \[ (e^{x+y} + 5) + (e^{x+y} + 6) y' = 0 \]

   d. \[ (e^{2x+y} + 2) + (e^{x+y} + 4) y' = 0 \]

   e. \[ (\cos(x + y) + 1) + (\cos(x + y) + 2) y' = 0 \]

   f. \[ (y/x + 6x) + (\ln x - 2)y' = 0 \]
2. Solve the following initial value problems
   
a. \((3x^2) + (5y^4)y' = 0\), \(y(1) = 1\).
   
b. \((x^2 + y) + (y^2 + x)y' = 0\), \(y(1) = 2\).

3. By finding an appropriate integrating factor (a function of \(x\)) solve the diff.eq
   
a. \(y' = e^{2x} + y - 1\).
   
b. \(2 + \frac{3y^2}{x}y' = 0\).

Dr. Z.’s Calc4 Homework assignment 7

Version of Sept. 26, 2013, #5 is optional

You can use a calculator for this assignment

1. Use the Euler method to find an approximate value for \(y(5)\) if \(y(x)\) is the solution of the initial value problem ode
   \(y' = x + y\), \(y(1) = 0\),
   using mesh-size \(h = 1\).

2. Use the Improved Euler method to find an approximate value for \(y(5)\) if \(y(x)\) is the solution of the initial value problem ode
   \(y' = x + y\), \(y(1) = 0\),
   using mesh-size \(h = 1\).

3. Use the Euler method to find an approximate value for \(y(1.1)\) if \(y(x)\) is the solution of the initial value problem ode
   \(y' = 4xy\), \(y(1) = 1\),
   using mesh-size \(h = 0.05\).

4. Use the Improved Euler method to find an approximate value for \(y(1.1)\) if \(y(x)\) is the solution of the initial value problem ode
   \(y' = 4xy\), \(y(1) = 1\),
using mesh-size $h = 0.05$.

5. [OPTIONAL] Use the RK4 to find an approximate value for $y(1.1)$ if $y(x)$ is the solution of the initial value problem ode

$$y' = 4xy \quad , \quad y(1) = 1$$

using mesh-size $h = 0.05$.

Dr. Z.’s Calc4 Homework assignment 8

1. Find the general solution of the following diff. eq.
   a. 
   $$y'' - 7y' + 12y = 0$$
   b. 
   $$y'' - 25y = 0$$
   c. 
   $$y'' + y' - 6y = 0$$
   d. 
   $$y'' - y' - 6y = 0$$
   e. 
   $$y'' + 10y' + 23y = 0$$
   f. 
   $$6y'' + 5y' - 4y = 0$$

2. Find the solution of the following initial value diff. eq.
   a. 
   $$y'' - 7y' + 12y = 0 \quad , \quad y(0) = 0 \quad , \quad y'(0) = -1$$
   b. 
   $$y'' - 25y = 0 \quad , \quad y(0) = 3 \quad , \quad y'(0) = -9$$
   c. 
   $$y'' - y' = 0 \quad , \quad y(0) = 3 \quad , \quad y'(0) = 2$$

Dr. Z.’s Calc4 Homework assignment 9
1. For each of the following diff.eq. initial value problems, and intervals, decide whether the theorem promises you that there is a unique solution. Explain!

a. \((t^3 - 1) y''(t) + \sin t y'(t) + (t^3 + 1) y(t) = e^t , \quad y(0) = 1 , \quad y'(0) = 3 ; \quad -2 < t < 2\)

b. \((t^3 + 1) y''(t) + \sin t y'(t) + (t^3 + 1) y(t) = e^t , \quad y(\frac{3}{2}) = 1 , \quad y'(\frac{3}{2}) = 3 ; \quad 1 < t < 2\)

c. \((t^3 - 8) y''(t) + \sin t y'(t) + (t^3 + 1) y(t) = e^t , \quad y(0) = 1 , \quad y'(0) = 3 ; \quad -3 < t < 1\)

d. \(t^2 y''(t) + \sin t y'(t) + (t^3 + 1) y(t) = e^t , \quad y(\frac{1}{2}) = 1 , \quad y'(\frac{1}{2}) = 3 ; \quad \frac{1}{1000} < t < 1\)

e. \(y''(t) + \csc t y'(t) + (t^3 + 1) y(t) = e^{t^2} , \quad y(0) = 1 , \quad y'(0) = 3 ; \quad -1 < t < 1\)

2. Find the largest open interval for which the solutions of the following diff.eqs. exist and are unique. Explain!

a. \((t - 1)(t - 4) y''(t) + \sin t y'(t) + \cos t y(t) = t , \quad y(0) = 1 , \quad y'(0) = 3\)

b. \((t + 100)(t - 200) y''(t) + \sin t y'(t) + \cos(e^t) y(t) = t^3 , \quad y(10) = 1 , \quad y'(10) = 3\)

c. \((t^2 + 1) y''(t) + \sin t y'(t) + \sin(e^{t^3}) y(t) = e^{-t^3} , \quad y(3) = -1 , \quad y'(3) = 13\)

d. \((t - 3)(t - 1)(t + 1)(t + 5) y''(t) + \sin t y'(t) + \cos(e^t) y(t) = t^3 , \quad y(0) = 1 , \quad y'(0) = 3\)

3. Find the Wronskian, \(W(f(t), g(t))\) of the following pair of functions.

a. \(f(t) = e^{t^2} , \quad g(t) = t e^{3t}\)

b. \(f(t) = e^{3t} , \quad g(t) = 3 e^{3t}\)

c. \(f(t) = e^{2t} , \quad g(t) = t e^{2t}\)

d. \(f(t) = t e^{2t} , \quad g(t) = t e^{3t}\)

4. Find the Wronskian (up to a constant in front) of any two solutions of the following diff.eq.

a. \(y''(t) + e^{-t} y'(t) + e^t y(t) = 0 .\)

b. \(t y''(t) + t \sin 3t y'(t) + e^t y(t) = 0 .\)
c. \[ y''(t) + 3y'(t) + 2y(t) = 0. \]

Dr. Z.’s Calc4 Homework assignment 10

1. Find the general solution to the following diff.eqs.
   a. \[ y''(t) - 2y'(t) + 2y(t) = 0 \]
   b. \[ y''(t) + 2y'(t) - 8y(t) = 0 \]
   c. \[ y''(t) + 2y'(t) + 2y(t) = 0 \]
   d. \[ y''(t) + 25y(t) = 0 \]
   e. \[ 4y''(t) + 16y'(t) + 25y(t) = 0 \]

2. Solve the following the initial value problems and for each state the nature of the oscillation (growing, steady, or decaying).
   a. \[ y''(t) + 4y(t) = 0, \quad y(0) = 0, \quad y'(0) = 1 \]
   b. \[ y''(t) + 4y'(t) + 5y(t) = 0, \quad y(0) = 1, \quad y'(0) = 0 \]
   c. \[ y''(t) - 2y'(t) + 5y(t) = 0, \quad y(\pi/2) = 1, \quad y'(\pi/2) = 2 \]
   d. \[ 4y''(t) + 4y'(t) + 5y(t) = 0, \quad y(0) = 3, \quad y'(0) = 1 \]
   e. \[ y''(t) + y(t) = 0, \quad y(\pi/3) = 2, \quad y'(\pi/3) = -4 \]

Dr. Z.’s Calc4 Homework assignment 11

Version of Oct. 7, 2013 (thanks to Samantha Heller)

1. Find the general solution to the following diff.eqs.
   a. \[ y''(t) - 6y'(t) + 9y(t) = 0 \]
   b. \[ y''(t) + 100y'(t) + 2500y(t) = 0 \]
   c. \[ y''(t) - 2\sqrt{2}y'(t) + 2y(t) = 0 \]
   d. \[ y''(t) + 2\sqrt{3}y'(t) + 3y(t) = 0 \]
25 \ y''(t) + 30 \ y'(t) + 9 \ y(t) = 0 \ .

2. Find the solution of the following initial value problem
a. 
\[ y''(t) + 4y'(t) + 4y(t) = 0 \ , \quad y(0) = 1 \ , \quad y'(0) = 0 \ . \]
b. 
\[ y''(t) + 2y'(t) + y(t) = 0 \ , \quad y(0) = 1 \ , \quad y'(0) = 2 \ . \]
c. 
\[ y''(t) - 6y'(t) + 9y(t) = 0 \ , \quad y(1) = 2e^3 \ , \quad y'(1) = 7e^3 \ . \]

3. Verify that the given solution \( y_1(t) \) is indeed a solution of the given diff.eq. Then Find a second solution of the given differential equation. Then write down the general solution.

a. 
\[ 3t^2 \ y''(t) - 12t \ y'(t) + 18y(t) = 0 \ , \quad t > 0 \ ; \quad y_1(t) = t^2 \ . \]
b. 
\[ t^2 \ y''(t) + 2t \ y'(t) - 2y(t) = 0 \ , \quad t > 0 \ ; \quad y_1(t) = t \ . \]
c. 
\[ x \ y''(x) - y'(x) + 4x^3y(x) = 0 \ , \quad x > 0 \ ; \quad y_1(x) = \sin x^2 \ . \]
d. 
\[ (x - 1) \ y''(x) - x \ y'(x) + y(x) = 0 \ , \quad x > 1 \ ; \quad y_1(x) = e^x \ . \]

Dr. Z.'s Calc4 Homework assignment 12

1. Find a particular solution to the following diff.eqs.

a. 
\[ y''(t) - 5y'(t) + 6y(t) = t \ . \]

b. 
\[ y''(t) - 5y'(t) + 6y(t) = te^t \ . \]

c. 
\[ y''(t) + y(t) = e^t \ . \]
d. \[ y''(t) + 2y'(t) + y(t) = 5 \ . \]

e. \[ y''(t) - 5y'(t) + 6y(t) = e^{3t} \ . \]

f. \[ y''(t) + y(t) = 3\sin 2t + 6\cos 2t \ . \]

g. \[ y''(t) + y(t) = \sin t \ . \]

1’. Find the general solution to the following diff.eqs.

a. \[ y''(t) - 5y'(t) + 6y(t) = t \ . \]

b. \[ y''(t) - 5y'(t) + 6y(t) = te^t \ . \]

c. \[ y''(t) + y(t) = e^t \ . \]

d. \[ y''(t) + 2y'(t) + y(t) = 5 \ . \]

e. \[ y''(t) - 5y'(t) + 6y(t) = e^{3t} \ . \]

f. \[ y''(t) + y(t) = 3\sin 2t + 6\cos 2t \ . \]

g. \[ y''(t) + y(t) = \sin t \ . \]

1”. Solve the following initial value problems

a. \[ y''(t) - 5y'(t) + 6y(t) = t \ , \quad y(0) = 1 \ , \ y'(0) = 1 \ . \]
b. \[ y''(t) - 5 y'(t) + 6 y(t) = te^t, \quad y(0) = 3, \quad y'(0) = 0 \]

c. \[ y''(t) + y(t) = e^t, \quad y(0) = 0, \quad y'(0) = 0 \]

d. \[ y''(t) + 2y'(t) + y(t) = 5, \quad y(0) = 5, \quad y'(0) = 1 \]

e. \[ y''(t) - 5y'(t) + 6y(t) = e^{3t}, \quad y(0) = 0, \quad y'(0) = 1 \]

f. \[ y''(t) + y(t) = 3 \sin 2t + 6 \cos 2t, \quad y(0) = 3, \quad y'(0) = 2 \]

g. \[ y''(t) + y(t) = \sin t, \quad y(0) = 1, \quad y'(0) = 1 \]

Dr. Z.'s Calc4 Homework assignment 13

1. Using Variation of Parameters, find a particular solution of
a. \[ y''(t) - y'(t) - 2y(t) = 2e^{-t} \]

b. \[ y''(t) - 5y'(t) + 6y(t) = e^{3t} \]

c. \[ y''(t) - 10y'(t) + 25y(t) = te^{5t} \]

2. Find the general solution of the following diff.eqs.

a. \[ y''(t) + y(t) = \tan t, \quad 0 < t < \pi/2 \]

b. \[ y''(t) - 4y'(t) + 4y(t) = \frac{e^{2t}}{4 + t^2} \]

3. Verify that the given functions are (independent) solutions of the corresponding homogeneous linear diff.eq., and find the general solution of the diff.eq.
\[ x^2 y''(x) - 2y(x) = 3x^2 + 1, \quad x > 0; \quad y_1(x) = x^2, \quad y_2(x) = \frac{1}{x}. \]

b. \[ x^2 y''(x) - x(x + 2)y'(x) + 2x^3 = x^2, \quad x > 0; \quad y_1(x) = x, \quad y_2(x) = xe^x. \]

**Dr. Z.'s Calc4 Homework assignment 14**

**Version of Oct. 30, 2013** (thanks to Xiao Cheng and Chris Grud)

1. For each of the following diff.eq. initial value problems, and intervals, decide whether the theorem promises you that there is a unique solution.

a. \[(t^3 + 1)y'''(t) + (\cos t) y''(t) + (t^2 + 1)y(t) = \sin t, \quad y(0) = 1, \quad y'(0) = 3, \quad y''(0) = 5; \quad -3 < t < 1\]

b. \[(t^3 + 1)y'''(t) + (\cos t) y''(t) + (t^2 + 1)y(t) = \sin t, \quad y(0) = 1, \quad y'(0) = 3, \quad y''(0) = 5; \quad -\frac{1}{3} < t < 3\]

c. \[y'''(t) + (\tan t) y''(t) + \sin t y(t) = \sin^2 t, \quad y(0.0001) = 1, \quad y'(0.0001) = 3, \quad y''(0.0001) = 5; \quad 0 < t < \pi/4\]

Note added Oct. 29, 2013 about 1c: Before the problem did not make sense, since the initial time, \(t_0 = 0\), was not in the interval. I thank Xiao Cheng for pointing it out.

d. \[y'''(t) + (\tan t) y''(t) + \sin t y(t) = \sin^2 t, \quad y(0.000000001) = 1, \quad y'(0.000000001) = 3, \quad y''(0.000000001) = 5; \quad 0 < t < 2\pi/3\]

Note added Oct. 30, 2013 about 1d: Before the problem did not make sense, since the initial time, \(t_0 = 0\), was not in the interval. I thank Chris Grud for pointing it out.

2. Find the largest open interval for which there is guaranteed to be a unique solution to

a. \[(t-1)(t-2)(t+3)(t+5)y^{(4)}(t) + (\sin t) y''(t) + (\sin^2 t) y(t) + (\cos t) y(t) = t^3, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = -1, \quad y'''(0) = 4, \quad 0 < t < 2\pi/3\]

b. \[(t^2 + 1)y^{(4)}(t) + (\sin t) y''(t) + (\sin^2 t) y(t) + (\cos t) y(t) = t^3, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = -1, \quad y'''(0) = 4, \quad 0 < t < 2\pi/3\]

c. \[(t^2 - 4)y^{(4)}(t) + (\sin t) y''(t) + (\sin^2 t) y(t) + \cos t y(t) = t^3, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = -1, \quad y'''(0) = 4, \quad 0 < t < 2\pi/3\]

3. Decide whether the following functions are linearly independent or linearly dependent. In the latter case find a linear relation among them.
a. $y_1(t) = 1 + e^t, y_2(t) = 2 + e^t, y_3(t) = 1 - e^t$

b. $y_1(t) = 1 + t^3, y_2(t) = 1 + t^4, y_3(t) = 2 + t^3 + t^4$

c. $y_1(t) = 1 + t^3, y_2(t) = 1 + t^4, y_3(t) = 3 + t^3 + t^4$

4.: Verify that the given functions are solutions of the diff.eq. and determine their Wronskian.

Note: (Added Oct. 26, 2014: thanks to Neha Bhat) When you ‘verify’ something, it may turn out to be false. If it is just say so. In these problem, if it so happens that not all proposed functions are solutions of the given diff.eq., you must say so. In that case, the second part, computing the Wronskian, is a futile excercise, that nevertheless makes sense, only it is irrelevant to the given diff. eq. .

a. $y'''(x) - 3y''(x) + 2y'(x) = 0 ; 1, e^x , e^{2x}$

b. $y'''(x) - y''(x) + y'(x) - y(x) = 0 ; 1, \sin x , \cos x$

c. $y'''(t) - 6y''(t) + 11y'(t) - 6y(t) = 0 ; e^t , e^{2t} , e^{3t}$

d. $x^3y'''(x) + x^2y''(x) - 2xy'(x) + 2y(x) = 0 ; x , \frac{1}{x} , x^2$

**Dr. Z.'s Calc4 Homework assignment 15**

1.: Find the general solution of

a. $y'''(x) + 3y''(x) - y'(x) - 3y(x) = 0$

b. $y^{(4)}(t) - 5y''(t) + 4y(t) = 0$

c. $y^{(4)}(t) - 8y''(t) - 9y(t) = 0$

d. $y^{(4)}(t) - y(t) = 0$

e. $y'''(t) - 6y''(t) + 12y'(t) - 8y(t) = 0$

f. $y'''(t) + 2y''(t) + y'(t) = 0$

1'.: Solve the initial value problems

a. $y'''(x) + 3y''(x) - y'(x) - 3y(x) = 0; \; y(0) = 1 \; , \; y'(0) = 0 \; , \; y''(0) = 0$
b. \( y^{(4)}(t) - 5y''(t) + 4y(t) = 0; \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = 0, \quad y'''(0) = 0 \)

c. \( y^{(4)}(t) - 8y''(t) - 9y(t) = 0; \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = 0, \quad y'''(0) = 0 \)

Dr. Z.’s Calc4 Homework assignment 16

1. Find a particular solution of the diff.eq.

a. \( y'''(t) - y''(t) + y'(t) - y(t) = e^{-t} \)

b. \( y'''(t) - y''(t) + y'(t) - y(t) = e^t \)

c. \( y'''(t) - y''(t) + y'(t) - y(t) = \sin 2t \)

d. \( y'''(t) - y''(t) + y'(t) - y(t) = \cos t \)

e. \( y'''(t) - y''(t) + y'(t) - y(t) = e^{-t} + e^t + \sin 2t + \cos t \)

(Hint: Don’t do it from scratch, use the answers for a,b,c,d).

f. \( y'''(t) + y'(t) = 1 + \cos t \)

2. Set-up but do not solve a template for a particular solution of

a. \( y^{(4)}(t) - 2y''(t) + y(t) = t^2 + e^t + e^{-t} + t^3 e^{2t} \)

b. \( y^{(4)}(t) + 2y''(t) + y(t) = 3 \cos t + t^2 + te^t \)

Dr. Z.’s Calc4 Homework assignment 17

1. Convert the following diff.eqs to a system of first-order diff.eqs

a. \( y^{(4)}(t) = \sqrt{(y'''(t) + t^3 + y'(t)y(t)) + \cos y(t) + e^{y'(t)}} \)

b. \( y^{(5)}(t) = ty''(t) + t^2 y(t) \)
2. Solve the initial value problem for the system, by using techniques for solving a single diff.eq. of higher order

a. 
\[ x'_1(t) = x_2(t) \quad , \quad x'_2(t) = -x_1(t) \; ; \; x_1(0) = 0 \; , \; x_2(0) = 1 \; . \]

b. 
\[ x'_1(t) = 3x_1(t) - 2x_2(t) \quad , \quad x'_2(t) = 2x_1(t) - 2x_2(t) \; ; \; x_1(0) = 3 \; , \; x_2(0) = \frac{1}{2} \; . \]

c. 
\[ x'_1(t) = -\frac{1}{2}x_1(t) + 2x_2(t) \quad , \quad x'_2(t) = -2x_1(t) - \frac{1}{2}x_2(t) \; ; \; x_1(0) = -2 \; , \; x_2(0) = 2 \; . \]

d. 
\[ x'_1(t) = x_3(t) \quad , \quad x'_2(t) = -x_1(t) - 1 \; , \quad x'_3(t) = x_1(t) + 1 \; ; \]
\[ x_1(0) = 3 \; , \; x_2(0) = 1 \; , \; x_3(0) = 0 \; . \]

3. If
\[ A = \begin{pmatrix} 2 & 3 & 4 \\ -2 & 1 & 3 \\ 1 & -1 & 2 \end{pmatrix} \quad , \quad B = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 0 & 1 \\ -1 & 0 & 2 \end{pmatrix} \]

Compute

a. \( AB \)

b. \( A^2 \)

c. \( AB - BA \).

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**Dr. Z.'s Calc4 Homework assignment 18**

1. Find the eigenvalues, and corresponding eigenvectors of the following 2 x 2 matrices.

a. 
\[ \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} \]

b. 
\[ \begin{pmatrix} 6 & -4 \\ 8 & -2 \end{pmatrix} \]
c. \[
\begin{pmatrix}
-2 & 1 \\
1 & -2
\end{pmatrix}
\]
d. \[
\begin{pmatrix}
1 & i \\
-i & 1
\end{pmatrix}
\]

2. Find the eigenvalues, and corresponding eigenvectors of the following $3 \times 3$ matrices.

a. \[
\begin{pmatrix}
3 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 2
\end{pmatrix}
\]
b. \[
\begin{pmatrix}
1 & 0 & 0 \\
2 & 1 & -2 \\
3 & 2 & 1
\end{pmatrix}
\]
c. \[
\begin{pmatrix}
3 & 2 & 2 \\
1 & 4 & 1 \\
-2 & -4 & -1
\end{pmatrix}
\]

3. Write the following system in matrix notation

\[
x_1'(t) = -t \, x_3(t) + t^3 \, x_4(t) - 2t \, x_2(t) + \frac{1}{t+1}
\]
\[
x_2'(t) = -x_3(t) - 3 \, x_1(t) + \cos^2 t
\]
\[
x_3'(t) = -9 \, x_2(t) + (\tan t) \, x_1(t) + 3t \, \sin^3 t \, x_4(t)
\]
\[
x_4'(t) = -x_4(t) + \tan^2 t \, x_1(t) + 6
\]
in matrix notation.

**Dr. Z.'s Calc4 Homework assignment 19**

1. Find the general solution of the systems

a. \[
x'(t) = \begin{pmatrix}
-4 & 2 \\
-15 & 7
\end{pmatrix} x(t)
\]
b. \[
x'(t) = \begin{pmatrix}
-3 & 2 \\
-15 & 8
\end{pmatrix} x(t)
\]
c. \[ x'(t) = \begin{pmatrix} 11 & -4 \\ 30 & -11 \end{pmatrix} x(t) \]

2.

Solve the initial value system (you can use the answers to Problem 1)

a. \[ x'(t) = \begin{pmatrix} -4 & 2 \\ -15 & 7 \end{pmatrix} x(t), \quad x(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \]

b. \[ x'(t) = \begin{pmatrix} -3 & 2 \\ -15 & 8 \end{pmatrix} x(t), \quad x(0) = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \]

c. \[ x'(t) = \begin{pmatrix} 11 & -4 \\ 30 & -11 \end{pmatrix} x(t), \quad x(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \]

Dr. Z.'s Calc4 Homework assignment 20

1. Find the general solution of the systems

a. \[ x'(t) = \begin{pmatrix} -3 & 2 \\ -1 & -1 \end{pmatrix} x(t) \]

b. \[ x'(t) = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} x(t) \]

c. \[ x'(t) = \begin{pmatrix} 1 & -1 \\ 5 & -3 \end{pmatrix} x(t) \]

d. \[ x'(t) = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{pmatrix} x(t) \]

2. Solve the initial value system. You may use the answers to Problem 1.

a. \[ x'(t) = \begin{pmatrix} -3 & 2 \\ -1 & -1 \end{pmatrix} x(t), \quad x(0) = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \]

b. \[ x'(t) = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} x(t), \quad x(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \]
c. \[ x'(t) = \begin{pmatrix} 1 & -1 \\ 5 & -3 \end{pmatrix} x(t) , \quad x(0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix} . \]

d. \[ x'(t) = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{pmatrix} x(t) , \quad x(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} . \]

Dr. Z.’s Calc4 Homework assignment 21

1. Find the general solution for each of the following systems

a. \[ x'(t) = \begin{pmatrix} 4 & -2 \\ 8 & -4 \end{pmatrix} x(t) . \]

b. \[ x'(t) = \begin{pmatrix} -3 & 5/2 \\ -5/2 & 2 \end{pmatrix} x(t) . \]

c. \[ x'(t) = \begin{pmatrix} 1 & -4/7 \end{pmatrix} x(t) . \]

2. Solve the initial value systems (You should use the answers to Problem 1).

a. \[ x'(t) = \begin{pmatrix} 4 & -2 \\ 8 & -4 \end{pmatrix} x(t) , \quad x(0) = \begin{pmatrix} -1 \\ 1 \end{pmatrix} . \]

b. \[ x'(t) = \begin{pmatrix} -3 & 5/2 \\ -5/2 & 2 \end{pmatrix} x(t) , \quad x(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} . \]

c. \[ x'(t) = \begin{pmatrix} 1 & -4/7 \end{pmatrix} x(t) , \quad x(0) = \begin{pmatrix} -6 \\ -4 \end{pmatrix} . \]

Dr. Z.’s Calc4 Homework assignment 22

1. Classify the critical point (0, 0) as to type, and determine whether it is is stable, asymptotically stable, or unstable, for the following systems.

a. \[ x'(t) = \begin{pmatrix} -6 & -10 \\ 5 & 8 \end{pmatrix} x(t) . \]
1. Determine the center of convergence and radius of convergence of the given power series
   
a. \[ \sum_{n=0}^{\infty} \frac{n^n}{3^n} x^n \]
   
b. \[ \sum_{n=0}^{\infty} \frac{2^n}{n^2} (x - 1)^n \]
   
c. \[ \sum_{n=0}^{\infty} \frac{2^n x^n}{n!} \]
   
d. \[ \sum_{n=0}^{\infty} \frac{(2x + 1)^n}{n^2} \]

2. Determine the Taylor series about the point \( x_0 \) for the given function. Also determine the radius of convergence
   
a. \( e^x, x_0 = 0 \)
   
b. \( \ln x, x_0 = 1 \)
c. \( \frac{1}{1-x}, \ x_0 = 0 \)

d. \( \frac{1}{1-x}, \ x_0 = 3 \)

Dr. Z.’s Calc4 Homework assignment 24

Problem 1. For the following:

(i) Seek power series solution of the given differential equation at \( x_0 = 0 \), find the recurrence relation.

(ii) Find the first four terms in each of two solutions \( y_1(x), y_2(x) \) (unless the series terminates sooner)

a. \( y''(x) - y(x) = 0 \)

b. \( y''(x) + y(x) = 0 \)

c. \( y''(x) - xy'(x) - y(x) = 0 \)

d. \( (1 - x)y''(x) + y(x) = 0 \)