Solutions to the Attendance Quiz # 10 for Dr. Z.’s Calc4 for Oct. 7, 2013

1. Find the general solution to the following diff.eq.

\[ y''(t) - 4y'(t) + 13y(t) = 0 \]

**Sol. to 1**: The characteristic equation is

\[ r^2 - 4r + 13 = 0 \]

Its roots are

\[
\frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 13}}{2 \cdot 1} = \frac{4 \pm \sqrt{16 - 52}}{2} = \frac{4 \pm \sqrt{-36}}{2} = \frac{4 \pm 6i}{2} = 2 \pm 3i.
\]

So \( \lambda = 2 \) and \( \mu = 3 \) and the general solution is

\[ y(t) = e^{2t}(c_1 \sin 3t + c_2 \cos 3t) \]

**Ans. to 1**: \( y(t) = e^{2t}(c_1 \sin 3t + c_2 \cos 3t) \), where \( c_1, c_2 \) are arbitrary constants.

**Comments**: Some people gave the answer

\[ y(t) = c_1 e^{(2+3i)t} + c_2 e^{(2-3i)t} \]

This is correct, but not in the real world. You would get at most half credit for that. The answer should be without imaginary stuff (i.e. no \( i \)).

Some people gave \( y(t) = c_1 e^{(2+3i)} + c_2 e^{(2-3i)} \) This is nonsense (without the \( t \)). You would get no points for that.

2. Solve the following the initial value problem and state the nature of the oscillation (growing, steady, or decaying).

\[ y''(t) + y(t) = 0 , \quad y(0) = 0 , \quad y'(0) = 1 \]

**Sol. to 2**: We first find the general solution. The characteristic equation is

\[ r^2 + 1 = 0 \]

So \( r^2 = -1 \) and the roots are \( 0 \pm \sqrt{-1} = \pm i = 0 \pm 1 \cdot i \). So \( \lambda = 0 \) and \( \mu = 1 \), and the general solution is

\[ y(t) = c_1 \sin t + c_2 \cos t \]

For future reference:

\[ y'(t) = c_1 \cos t - c_2 \sin t \].
Plug-in $t = 0$:

\[ y(0) = c_1 \sin 0 + c_2 \cos 0 = c_2 \]
\[ y'(0) = c_1 \cos 0 - c_2 \sin 0 = c_1 \]

Since $y(0) = 0$ and $y'(0) = 1$ we get the equations

\[ 0 = c_2 \quad , \quad 1 = c_1 \]

whose solutions are $c_1 = 1$ and $c_2 = 0$. Going back to the general solution, we have the specific solution

\[ y(t) = 1 \cdot \sin t + 0 \cdot \cos t = \sin t \]

Since $\lambda = 0$ the solution is steady.

\textbf{Sol. to 2}: $y(t) = \sin t$. It is a steady solution.