1. Find the general solution to the following diff. eq.

\[ y''(t) + 10y'(t) + 25y(t) = 0 \, . \]

**Sol. to 1:** The characteristic equation is:

\[ r^2 + 10r + 25 = 0 \, . \]

Factorizing

\[ (r + 5)^2 = 0 \, . \]

So we have a **double root** \( r = -5 \). The general solution is

\[ y(t) = e^{-5t}(c_1 + c_2t) \, . \]

**Ans. to 1:** \( y(t) = e^{-5t}(c_1 + c_2t) \), where \( c_1, c_2 \) are **arbitrary constants**.

2. Find the solution of the following initial value problem

\[ y''(t) - 2y'(t) + y(t) = 0 \, , \, y(0) = 1 \, , \, y'(0) = 0 \, . \]

**Sol. to 2:** The characteristic equation is:

\[ r^2 - 2r + 1 = 0 \, . \]

Factorizing

\[ (r - 1)^2 = 0 \, . \]

So we have a **double root** \( r = 1 \). The general solution is

\[ y(t) = e^t(c_1 + c_2t) \, . \]

Now

\[ y'(t) = (e^t)'(c_1 + c_2t) + e^t(c_1 + c_2t)' = e^t(c_1 + c_2t) + c_2e^t \, . \]

Plugging in \( t = 0 \), we get

\[ y(0) = c_1 \, , \, y'(0) = c_1 + c_2 \, . \]

From the data of the problem \( y(0) = 1 \, , \, y'(0) = 0 \). So

\[ c_1 = 1 \, , \, c_1 + c_2 = 0 \, . \]

Solving this system, we get

\[ c_1 = 1 \, , \, c_2 = -1 \, . \]

Going back to the general solution:

\[ y(t) = e^t(1 - t) \, . \]

**Ans. to 2:** \( y(t) = e^t(1 - t) \).