1. Solve the initial value problem

\[ y'(t) - 3y(t) = e^{3t}, \quad y(0) = 1. \]

**Sol. of 1.:** The coefficient of \( y'(t) \) is already 1 so we don’t have to do any preprocessing.

The coefficient of \( y(t) \) is \(-3\). So

\[ p(t) = -3. \]

Integrating gives \( \int p(t) \, dt = \int (-3) \, dt = -3t \). (we don’t bother with the \(+C\)). So the Integrating factor, \( I(t) \) is

\[ I(t) = e^{\int p(t) \, dt} = e^{-3t}. \]

The right side, \( q(t) \) is \( e^{3t} \). Using the “canned formula”

\[ y(t) = \frac{\int I(t)q(t) \, dt}{I(t)}, \]

we have

\[ y(t) = \frac{\int e^{-3t}e^{3t} \, dt}{e^{-3t}}, \]

\[ = \frac{\int 1 \, dt}{e^{-3t}} = \frac{t + C}{e^{-3t}}. \]

Now it is time to find \( C \) (a number!) . Since \( y(0) = 1 \), we have

\[ 1 = \frac{0 + C}{e^{-3 \cdot 0}}. \]

So

\[ C = 1. \]

Going back to the general solution, we get the final answer

\[ y(t) = \frac{t + 1}{e^{-3t}}. \]

Finally, doing the simple algebra, we get

**Ans. to 1.:** \( y(t) = (t + 1)e^{3t} \).

**Comment:** About \( 60\% \) got it perfectly. Most other people were on the right track, but messed up the algebra at the end. One student got \( C = e^{-t} \). This is nonsense. \( C \) is a constant, it can’t depend on \( t \).

2. Find the general solution of \( y'(t) - y(t) = e^{2t} \).
Sol. of 2: Again $y'(t)$ already has coefficient 1. $p(t) = -1$, $\int p(t) \, dt = \int (-1) \, dt = -t$.

So $I(t) = e^{-t}$. Also $q(t) = e^{2t}$

\[
y(t) = \frac{\int I(t)q(t) \, dt}{I(t)} = \frac{\int e^{-t}e^{2t} \, dt}{e^{-t}} = \frac{\int e^{t} \, dt}{e^{-t}} = e^{t} + C \cdot \frac{e^{t}}{e^{-t}} = e^{2t} + Ce^{t}.
\]

Ans. to 2: $y(t) = e^{2t} + Ce^{t}$, where $C$ is an arbitrary constant.