Solutions to Attendance Quiz # 6 for Dr. Z.’s Calc4 for Sept. 23, 2013

1. For the following first-order diff.eq., decide whether or not it is exact. If it is, solve it!

\[(3x^2y^2 + 4x^3y) + (2x^3y + x^4)y' = 0\]

**Sol. to 1:** Here

\[M = 3x^2y^2 + 4x^3y, \quad N = 2x^3y + x^4.\]

\[M_y = 6x^2y + 4x^3, \quad N_x = 6x^2y + 4x^3.\]

Since \(M_y = N_x\) this diff.eq. is **exact**, and we must go on.

\[F = \int M \, dx = (3x^2y^2 + 4x^3y) \, dx = x^3y^2 + x^4y + \phi(y),\]

where \(\phi(y)\) is **to be determined**.

Now we use \(F_y = N\). Using our tentative \(F\) we have

\[F_y = \frac{\partial}{\partial y} (x^3y^2 + x^4y + \phi(y)) = 2x^3y + x^4 + \phi'(y).\]

Setting it equal to \(N\),

\[2x^3y + x^4 + \phi'(y) = 2x^3y + x^4.\]

Using algebra, we get \(\phi'(y) = 0\) so \(\phi(y) = 0 + C\). Going back to \(F\) above we get

\[F = x^3y^2 + x^4y + C.\]

**Note:** THIS IS NOT THE FINAL ANSWER. Some people left it like that. The final answer is of the format

\[F = C,\]

So we have

\[x^3y^2 + x^4y + C = C.\]

But \(C - C = C\) (an arbitrary constant minus an arbitrary constant is yet another arbitrary constant), so the final answer is

**Ans. to 1:**

\[x^3y^2 + x^4y = C.\]

\((C\ \text{an arbitrary constant}).\)
Note: It is OK to take $\phi(y) = 0$ above rather than $\phi(y) = C$, since at the end of the day we have a $C$ on the right side.

Comment: About half of the students got it completely.

2. Solve the following initial value problem

$$(2x + y) + (x + 2y)y' = 0 \quad , \quad y(1) = 3$$

**Sol. to 2:** $M = 2x + y$, $N = x + 2y$. $M_y = 1$, $N_x = 1$. So $M_y = N_x$ and our diff.eq. is **exact**. The next step is to find $F$.

$$F = \int M \, dx = (2x + y) \, dx = x^2 + yx + \phi(y)$$

Where $\phi(y)$ is **to be determined**.

Next:

$$F_y = \frac{\partial}{\partial y} (x^2 + yx + \phi(y)) = x + \phi'(y)$$

Setting this equal to $N$, we get the equation

$$x + \phi'(y) = x + 2y$$

Using algebra:

$$\phi'(y) = 2y$$

**Warning:** If at this step you have $x$ showing up on the right side it is a **symptom** that you messed up the algebra and/or the calculus. DO NOT CONTINUE. It is much better to realize that you messed up then to go on.

Intergarting with respect to $y$:

$$\phi(y) = \int 2y \, dy = y^2$$

(Note: no need for the $+C$ in this step).

Going back to the $F$ above

$$F = x^2 + yx + \phi(y) = x^2 + yx + y^2$$

This is **not** the final answer.

The general solution of the diff.eq. is $F = C$, so in this problem, it is

$$x^2 + yx + y^2 = C$$
Finally, since in this problem you were given the initial condition $y(1) = 3$ you plug-in $x = 1, y = 3$ getting,

$$1^2 + 1 \cdot 3 + 3^2 = C$$

$$1 + 3 + 9 = C$$

So $C = 13$. Going back to the general solution, we have:

**Ans. to 2:**

$$x^2 + yx + y^2 = 13$$

**Note:** The answer is a function given in implicit format. In this example, you can use algebra (the quadratic equation), to get $y$ as an explicit function of $x$. Don’t bother! Usually it is not possible (or very hard). The expected format for these diff.eqs. is always implicit format, i.e. $F(x, y) = C$, rather then $y = f(x)$. 