1. Find the general solution of the following diff. eq.

\[ y'' + y' - 20y = 0 \ . \]

**Sol. to 1:** The characteristic equation is

\[ r^2 + r - 20 = 0 \ . \]

Factoring

\[ (r + 5)(r - 4) = 0 \ . \]

The roots are \( r_1 = -5, r_2 = 4 \) so the general solution is

\[ y(t) = c_1 e^{-5t} + c_2 e^{4t} \ . \]

**Ans. to 1:** \( y(t) = c_1 e^{-5t} + c_2 e^{4t} \), where \( c_1 \) and \( c_2 \) are arbitrary constants.

2. Find the solution of the following initial value diff. eq.

\[ y'' - 3y' + 2y = 0 \ , \ y(0) = 2 \ , \ y'(0) = 3 \ . \]

**Sol. to 2:**

The characteristic equation is

\[ r^2 - 3r + 2 = 0 \ . \]

Factoring

\[ (r - 1)(r - 2) = 0 \ . \]

The roots are \( r_1 = 1, r_2 = 2 \) so the general solution is

\[ y(t) = c_1 e^t + c_2 e^{2t} \ . \]

Now we take the derivative

\[ y'(t) = c_1 e^t + 2c_2 e^{2t} \ . \]

Now we plug-in \( t = 0 \):

\[ y(0) = c_1 e^0 + c_2 e^{2\cdot0} = c_1 + c_2 \ . \]

\[ y'(0) = c_1 e^0 + 2c_2 e^{2\cdot0} = c_1 + 2c_2 \ . \]

But the problem tells us that \( y(0) = 2 \) and \( y'(0) = 3 \), so we have to solve the system of two equations and two unknowns:

\[ c_1 + c_2 = 2 \ , \ c_1 + 2c_2 = 3 \ . \]

The second equation minus the first one tells us that \( c_2 = 1 \) and plugging into the first (or second) equation, gives \( c_2 = 1 \). Going back to the general solution, we have

**Ans. to 2:** \( y(t) = e^t + e^{2t} \ . \)