1. Solve:

\[ u_{xx} + u_{yy} = 0 \quad 0 < x < \pi \quad 0 < y < 1 \]

Subject to

\[ u(0, y) = 0 \quad u(\pi, y) = 0 \quad 0 < y < 1 \]
\[ u(x, 0) = 0 \quad u(x, 1) = f(x) \quad 0 < x < \pi \]

2. Solve:

\[ u_{xx} + u_{yy} = 0 \quad 0 < x < \pi \quad 0 < y < 1 \]

Subject to

\[ u(0, y) = 0 \quad u(\pi, y) = 0 \quad 0 < y < 1 \]
\[ u_y(x, 0) = 0 \quad u(x, 1) = f(x) \quad 0 < x < \pi \]

3. Solve:

\[ u_{xx} + u_{yy} = 0 \quad 0 < x < \pi \quad 0 < y < 1 \]

Subject to

\[ u_x(0, y) = 0 \quad u_x(\pi, y) = 0 \quad 0 < y < 1 \]
\[ u_y(x, 0) = 0 \quad u(x, 1) = f(x) \quad 0 < x < \pi \]

4. Explain how you would solve the following boundary-value pde problem

\[ u_{xx} + u_{yy} = 0 \quad 0 < x < 3 \quad 0 < y < 4 \]
\[ u(0, y) = y^3 \quad u(3, y) = \cos 7y \quad 0 < y < 4 \]
\[ u(x, 0) = x^3 \quad u(x, 4) = e^x \quad 0 < x < 3 \]

By breaking it up into two simpler problems. **Do not solve these problems.**