Solutions to the Attendance Quiz 0

1. Approximate, with mesh-size $h = 1$, the solution of the boundary-value problem

$$u_{xx} + u_{yy} = 0, \quad 0 < x < 2, \quad 0 < y < 2;$$

subject to the boundary conditions

$$u(0, y) = 2, \quad 0 < y < 2; \quad u(2, y) = 3, \quad 0 < y < 2;$$

$$u(x, 0) = 1, \quad 0 < x < 2; \quad u(x, 2) = x, \quad 0 < x < 2.$$

**Sol.** The boundary points are

(1,0) (that lies on the **bottom** side), and $u_{10} = u(1,0)$

(0,1) (that lies on the **left** side), and $u_{01} = u(0,1)$

(1,2) (that lies on the **top** side), and $u_{12} = u(1,2)$

(2,1) (that lies on the **right** side), and $u_{21} = u(2,1)$

Using the boundary conditions we have

$$u_{10} = u(1,0) = 1$$

$$u_{01} = u(0,1) = 2$$

$$u_{12} = u(1,2) = 1$$

$$u_{21} = u(2,1) = 3$$

There is only one interior point (1,1) and its value there is called $u_{1,1}$. It gives rise to the equation

$$u_{11} = \frac{u_{10} + u_{01} + u_{12} + u_{21}}{4}$$

in the one unknown, $u_{11}$.

Using the above values for the boundary points we get:

$$u_{11} = \frac{1 + 2 + 1 + 3}{4} = \frac{7}{4}.$$

**Ans.** $u(1,1) \approx \frac{7}{4}$.

**Comments:**
1. About 80% of the students got it perfectly. Those who didn’t, please go over the handout and this solution and understand it really well.

2. Quite a few people did it almost perfectly, but they wrote \( u(1, 1) = \frac{7}{4} \), instead of the correct: \( u(1, 1) \approx \frac{7}{4} \). This numerical method (called the method of Finite Differences) only gives you approximations, and since here \( h \) is so big, this happens to be a very bad approximation, and the point of the problem is to teach you the method, but in real life it is done by computers, with a much smaller \( h \) (e.g. \( h = 0.1 \) or even \( h = 0.01 \) and then the approximations are very good.

3. Some people took \( u(1, 2) = x \) and got an expression for \( u(1, 1) \) that involves \( x \). This is GARBAGE! \( u(1, 1) \) equals (or rather approximately equals) a NUMBER not an expression in \( x \). They got mixed up because the boundary values on all the other sides were constants and only on \( y = 2 \) it was an expression in \( x \): \( u(x, 2) = x \). But the right thing to do is to realize that at the point \( (1, 2) \), the \( x \)-coordinate equals 1, so you plug-in \( x = 1 \) into the expression describing \( u(x, 2) \), namely \( x \), and get \( u(1, 2) = 1 \).