1. Find all the eigenvalues of the matrix
\[
\begin{pmatrix}
-3 & -4 \\
12 & 11
\end{pmatrix}
\]
and determine a basis for each eigenspace.

**Sol.:** The **Characteristic matrix** is
\[
\begin{pmatrix}
-3 - \lambda & -4 \\
12 & 11 - \lambda
\end{pmatrix}
\].

Taking its **determinant**, we have
\[
\det \begin{pmatrix} -3 - \lambda & -4 \\ 12 & 11 - \lambda \end{pmatrix} = (-3-\lambda)(11-\lambda)-(-4)(12) = (\lambda+3)(\lambda-11)+48 = \lambda^2-8\lambda-33+48 = \lambda^2-8\lambda+15
\]
So the **characteristic equation** is:
\[
\lambda^2 - 8\lambda + 15 = 0
\]
Factorizing we get
\[
(\lambda - 3)(\lambda - 5) = 0
\]
So the **eigenvalues** are \(\lambda = 3\) and \(\lambda = 5\).

For each of them we must still find a basis for the **eigenspace**.

For \(\lambda = 3\), we have to find vector(s) \([a \\ b]\\), such that
\[
\begin{pmatrix} -3 & -4 \\ 12 & 11 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = 3 \begin{pmatrix} a \\ b \end{pmatrix}
\].
In everyday notation:
\[
-3a - 4b = 3a , \quad 12a + 11b = 3b
\]
Rearranging:
\[
-6a - 4b = 0 , \quad 12a + 8b = 0
\]
The second equation is twice the first one, so we can ignore it, and we get \(b = -\frac{3}{2}a\). To make it nice, we can take \(a\) to be any non-zero number. Taking \(a = 2\) gives \(b = -3\), so a basis vector is \([2 \\ -3]\\).

For \(\lambda = 5\), we have to find vector(s) \([a \\ b]\\), such that
\[
\begin{pmatrix} -3 & -4 \\ 12 & 11 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = 5 \begin{pmatrix} a \\ b \end{pmatrix}
\].
In everyday notation:

\[-3a - 4b = 5a \quad 12a + 11b = 5b \quad .\]

Rearranging:

\[-8a - 4b = 0 \quad 12a + 6b = 0 \quad .\]

The second equation is a multiple of the first one, so we can ignore it, and we get

\[b = -\frac{5}{4}a = -2a.\]

To make it nice, we can take \(a\) to be any non-zero number. Taking \(a = 1\) gives \(b = -2\), so a basis vector is \([1 -2]\).

\textbf{Ans.} The eigenvalues are \(\lambda = 3\) and a basis for its eigenspace is \([\begin{bmatrix} 2 \\ -3 \end{bmatrix}]\), and \(\lambda = 5\) and a basis for its eigenspace is \([\begin{bmatrix} 1 \\ -2 \end{bmatrix}]\).