1. Compute (using Lecture 4’s method!) $\mathcal{L}\{t^3e^{-2t}\}$.

**Sol. to 1:** We are supposed to use the formula

$$\mathcal{L}\{t^n f(t)\} = (-1)^n F^{(n)}(s)$$

where $F(s) = \mathcal{L}\{f(t)\}$.

Here $f(t) = e^{-2t}$ so $F(s) = \frac{1}{s+2} = (s+2)^{-1}$. We have $F'(s) = (-1)(s+2)^{-2}$, $F''(s) = (-1)(-2)(s+2)^{-3}$, and $F'''(s) = (-1)(-2)(-3)(s+2)^{-4} = -\frac{6}{(s+2)^4}$. Hence

$$\mathcal{L}\{t^3e^{-2t}\} = \frac{6}{(s+2)^4}$$

**Ans. to 1:** $\frac{6}{(s+2)^4}$.

2. Solve the IVP

$$y' - 3y = \delta(t-2) \quad , \quad y(0) = 1$$

**Sol. to 2:** Apply $\mathcal{L}$, we get

$$\mathcal{L}\{y'\} - 3\mathcal{L}\{y\} = \{\delta(t-2)\} = e^{-2s}$$

Putting, as usual $\mathcal{L}\{y\} = Y$, we have

$$sY - y(0) - 3Y = e^{-2s}$$

But, since $y(0) = 1$, we have

$$sY - 1 - 3Y = e^{-2s}$$

Solving for $Y$:

$$(s - 3)Y = 1 + e^{-2s}$$

dividing by $(s - 3)$:

$$Y = \frac{1}{s - 3} + \frac{e^{-2s}}{s - 3}$$

Since $\mathcal{L}^{-1}\{\frac{1}{s - 3}\} = e^{3t}$, we get,

**Ans. to 2:**

$$y(t) = e^{3t} + e^{3(t-2)}U(t-2)$$