1. (5 points) Find the first two coefficients of the Fourier-Legendre expansion of 

\[ f(x) = \begin{cases} 
2, & \text{if } -1 < x < 0; \\
-1, & \text{if } 0 \leq x < 1.
\end{cases} \]

**Sol.** \( P_0(x) = 1, P_1(x) = x, P_2(x) = \frac{1}{2}(3x^2 - 1) \). The general formula is 

\[ c_n = \frac{2n + 1}{2} \int_{-1}^{1} f(x)P_n(x) \, dx. \]

So:

\[ c_0 = \frac{2 \cdot 0 + 1}{2} \int_{-1}^{1} f(x)P_0(x) = \frac{1}{2} \int_{-1}^{1} f(x) = \frac{1}{2} \int_{-1}^{0} 2 + \frac{1}{2} \int_{-1}^{1} (-1) = \frac{1}{2} (2 - 1) = \frac{1}{2} \]

\[ c_1 = \frac{2 \cdot 1 + 1}{2} \int_{-1}^{1} f(x)P_1(x) = \frac{3}{2} \int_{-1}^{1} f(x) \cdot x = \frac{3}{2} \int_{-1}^{0} 2x + \frac{3}{2} \int_{0}^{1} (-1)(x) \]

\[ = \frac{3}{2} x^2 \bigg|_{-1}^{0} + \frac{3}{2} \left( \frac{-x^2}{2} \right)_{0}^{1} = \frac{3}{2} (0^2 - (-1)^2) + \frac{3}{2} \left( \frac{-1 - 0}{2} \right) = \frac{3}{2} - \frac{3}{4} = \frac{9}{4}. \]

**Ans. to 1:** The first two coefficients are \( c_0 = \frac{1}{2}, c_1 = -\frac{9}{4} \).

2. (5 points) Find product solutions, if possible, to the partial differential equation 

\[ \frac{\partial u}{\partial x} = 25 \frac{\partial u}{\partial y}. \]

**Sol.** Try \( u(x, y) = X(x)Y(y) \). So

\[ X'(x)Y(y) = 25X(x)Y'(y). \]

Divided both sides by \( X(x)Y(y) \):

\[ \frac{X'(x)}{X(x)} = 25 \frac{Y'(y)}{Y(y)}. \]

The left does not depend on \( y \), and the right does not depend on \( x \), and they are equal to each other, so neither depends on \( x \) or \( y \), so they are both equal to a number, let’s call it \( k \). We have traded one pde with two odes:

\[ \frac{X'(x)}{X(x)} = k \]

\[ 25 \frac{Y'(y)}{Y(y)} = k. \]

These are

\[ X'(x) - kX(x) = 0, \]
\[
Y'(y) - \left(\frac{k}{25}\right)Y(y) = 0.
\]

The general solutions are
\[
X(x) = c_1 e^{kx},
\]
\[
Y(y) = c_2 e^{\left(\frac{k}{25}\right)y}.
\]

So
\[
u(x, y) = (c_1 e^{kx})(c_2 e^{\left(\frac{k}{25}\right)y}) = (c_1 c_2) e^{kx} e^{\left(\frac{k}{25}\right)y}.
\]

Renaming \(c_1 c_2, C\), and simplifying we get
\[
u(x, y) = Ce^{kx + \left(\frac{k}{25}\right)y}.
\]

**Ans. to 2:** \(u(x, y) = Ce^{kx + \left(\frac{k}{25}\right)y}\), where \(C\) is an arbitrary constant.