1. Here \( f(x) = 5x^2 - 4x \) the **difference quotient** in general is

\[
\frac{f(x + \Delta x) - f(x)}{\Delta x}
\]

In this problem it is:

\[
\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{[5(x + \Delta x)^2 - 4(x + \Delta x)] - [5x^2 - 4x]}{\Delta x}
\]

Using the famous \((a + b)^2 = a^2 + 2ab + b^2\) this equals

\[
\frac{5x^2 + 10(\Delta x)x + 5(\Delta x)^2 - 4x - 4(\Delta x) - 5x^2 + 4x}{\Delta x}.
\]

Simplifying, this equals:

\[
\frac{10(\Delta x)x + 5(\Delta x)^2 - 4(\Delta x)}{\Delta x}.
\]

Factoring the top, this equals:

\[
\frac{(\Delta x)(10x + 5(\Delta x) - 4)}{\Delta x}.
\]

Cancelling out \(\Delta x\) from top and bottom gives that this equals

\[
10x - 4 + 5\Delta x.
\]

**Ans. to 1a:** The difference quotient is \(2x - 4 + 5\Delta x\).

(b) The derivative is the limit as \(\Delta x \to 0\).

\[
\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} 10x - 4 + 5\Delta x = 10x - 4 + 5 \cdot 0 = 10x - 4
\]

**Ans. to 1b:** The derivative \(dy/dx\) is \(10x - 4\).

**Sol. to 1c:** Since \(f'(x) = 10x - 4\), we have \(f'(2) = 10 \cdot 2 - 4 = 20 - 4 = 16\) and \(f'(3) = 10 \cdot 3 - 4 = 30 - 4 = 26\) and

2. Using the famous formula, \((a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3\) we can simplify \(q\)

\[
q = \frac{(v + 2)^3 - 8}{v} = \frac{v^3 + 3v^2(2) + 3v(2^2) + 2^3 - 8}{v} = \frac{v^3 + 6v^2 + 12v}{v} = \frac{(v^2 + 6v + 12)v}{v} = v^2 + 6v + 12.
\]

This is a nice polynomial in \(v\) so to find the limit at a point, just **plug-it in**

a. \(\lim_{v \to 0} q = 0^2 + 6 \cdot 0 + 12 = 12\)
b. \( \lim_{v \to 2} q = 2^2 + 6 \cdot 2 + 12 = 4 + 12 + 12 = 28 \)

c. \( \lim_{v \to a} q = a^2 + 6 \cdot a + 12 = 4 + 12 + 12 = 28 \)

3. We use the constant multiple rule (you can always take a constant out of the differentiation) and the famous rule
\[
\frac{d}{dx} x^n = nx^{n-1}
\]
a. 
\[
\frac{d}{dx} (-x^{-4}) = -\left(\frac{d}{dx} x^{-4}\right) = -(-4)x^{-5} = 4x^{-5}.
\]
b. 
\[
\frac{d}{du} (au^b) = a\left(\frac{d}{du} u^b\right) = ab u^{b-1}.
\]
c. 
\[
\frac{d}{dx} cx^2 = c\left(\frac{d}{dx} x^2\right) = 2cx.
\]
d. 
\[
\frac{d}{dx} (7x^{1/3}) = 7\left(\frac{d}{dx} x^{1/3}\right) = 7 \cdot \frac{1}{3}x^{-2/3} = \frac{7}{3}x^{-2/3}.
\]

4. The product rule is \((f(x)g(x))' = f'(x)g(x) + f(x)g'(x)\).
a. \((9x^2 - 1)(3x + 1)' = (9x^2 - 1)'(3x + 1) + (9x^2 - 1)(3x + 1)' = (18x)(3x + 1) + (9x^2 - 1)3 = 54x^2 + 18x + 27x^2 - 3 = 81x^2 + 18x - 3\)
b. \((ax-b)(cx^2)' = (ax-b)'(cx^2) + (ax-b)(cx^2)' = (a)(cx^2) + (ax-b)(2cx) = acx^2 + 2acx^2 - 2bcx = 3acx^2 - 2bcx\)
c. 
\[
[(2-3x)(1+x)(x+2)]' = (2-3x)'(1+x)(x+2) + (2-3x)(1+x)'(x+2) + (2-3x)(1+x)(x+2)' = (-3)(1+x)(x+2) + (2-3x)(1)(2-3x)(1+x) = -3(1+x)(x+2) + (2-3x)(x+2) + (2-3x)(1+x) = -3(1+x)(x+2) + (2-3x)(x+2) + (2-3x)(1+x).
\]
d. 
\[
[(x^2+3)x^{-1}]' = (x^2+3)'(x^{-1}) + (x^2+3)(x^{-1})' = (2x)(x^{-1}) + (x^2+3)(-1)x^{-2} = -2 - (1 + 3x^{-2}) = 1 - 3x^{-2}.
\]

5. For (a) and (b) it is better to first simplify, and then differentiate. For (c) and (d) we need the quotient rule.
a. 
\[
((x^2+3)/x)' = (x+3/x)' = (x+3x^{-1})' = x' + 3(x^{-1})' = 1 + 3(-1)x^{-2} = 1 - \frac{3}{x^2}.
\]
b. 

\[ ((x + 7)/x)' = (1 + 7/x)' = (1 + 7x^{-1})' = 1' + 7(x^{-1})' = 0 + 7(-1)x^{-2} = -\frac{7}{x^2}. \]

The quotient rule is:

\[ \left( \frac{f(x)}{g(x)} \right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}. \]

c. 

\[ \left( \frac{4x}{x + 5} \right)' = \frac{(4x)'(x + 5) - 4x(x + 5)'}{(x + 5)^2} = \frac{4(x + 5) - 4x(1)}{(x + 5)^2} = \frac{4x + 20 - 4x}{(x + 5)^2} = \frac{20}{(x + 5)^2}. \]

d. 

\[ \left( \frac{ax^2 + b}{cx + d} \right)' = \frac{(ax^2 + b)'(cx + d) - (ax^2 + b)(cx + d)'}{(cx + d)^2} = \frac{(2ax)(cx + d) - (ax^2 + b)(c)}{(cx + d)^2} = \frac{2acx^2 + 2adx - acx^2 - bc}{(cx + d)^2} = \frac{acx^2 + 2adx - bc}{(cx + d)^2}. \]

6. The chain rule says

\[ \frac{dw}{dx} = \frac{dw}{dy} \cdot \frac{dy}{dx}. \]

Here

\[ \frac{dw}{dy} = 2ay, \quad \frac{dy}{dx} = 2bx. \]

Hence

\[ \frac{dw}{dx} = (2ay)(2bx) = 4abxy. \]

Finally we replace \( y \) by what it stands for in terms of \( x \), getting

\[ \frac{dw}{dx} = 4abxy = 2abx(bx^2 + cx) = 2abx^2(bx + c). \]

7. 

\[ ((16x + 3)^{-2})' = (-2)((16x + 3)^{-3}) \cdot (16x + 3)' = (-2)((16x + 3)^{-3}) \cdot (16) = -32(16x + 3)^{-3}. \]

By the quotient rule:

\[ \left( \frac{1}{(16x + 3)^2} \right)' = \frac{1'(16x + 3)^2 - 1((16x + 3)^2)'}{(16x + 3)^4} = \frac{0 \cdot (16x + 3)^2 - 1((2)(16x + 3))(16)}{(16x + 3)^4} = -\frac{32}{(16x + 3)^3} = -32(16x + 3)^{-3}. \]

The answers are identical because God (or Newton) said so!
8. 

a. When $x$ is VERY BIG $-7x + 5$ and $-1$ are insignificant compared to $2x^2$ and $x$ respectively, hence it is possible to only retain the leading powers at the top and bottom.

$$\lim_{x \to \infty} \frac{x - 1}{2x^2 - 7x + 5} = \lim_{x \to \infty} \frac{x}{2x^2} = \lim_{x \to \infty} \frac{1}{2x} = \frac{1}{2\infty} = 0.$$

Ans. to 8a: 0.

Since the top and bottom happen to be 0 at $x = 1$, we can use L'Hôpital's rule:

$$\lim_{x \to 1} \frac{x - 1}{2x^2 - 7x + 5} = \lim_{x \to 1} \frac{(x - 1)'}{(2x^2 - 7x + 5)'} = \lim_{x \to 1} \frac{1}{4x - 7} = \frac{1}{4 \cdot 1 - 7} = -\frac{1}{3}.$$

Ans. to 8b: $-\frac{1}{3}$.

Comment: We can also do it only using algebra

$$\lim_{x \to 1} \frac{x - 1}{2x^2 - 7x + 5} = \lim_{x \to 1} \frac{x - 1}{(x - 1)(2x - 5)} = \lim_{x \to 1} \frac{1}{2x - 5} = \frac{1}{2 \cdot 1 - 5} = -\frac{1}{3}.$$

9.

First way: Using algebra, $f(x) = x + 1$ when $x \neq 1$. Since $f(1) = 2$ the function given is a complicated way of writing down $f(x) = x + 1$ that is well-known to be continuous.

Second Way: $f(x)$ is continuous when $x \neq 1$ (since it is a quotient of two continuous functions, and the denominator does not vanish). The limit of $f(x)$ as $x$ goes to 1 happens to be 2 (either by L'Hôpital or using algebra). Since the limit of $f(x)$ as $x$ goes to 1 equals $f(1)$, it is also continuous at $x = 1$, hence it is continuous over all the real line.

10. By the product and chain rules:

$$f'(x) = \left(\frac{1}{2}(x^2 + 4x - 9)^3x^{-2}\right)' = \frac{1}{2}((x^2 + 4x - 9)^3x^{-2})' = \frac{1}{2}[(x^2 + 4x - 9)^3]'x^{-2} + ((x^2 + 4x - 9)^3)(x^{-2})' = \frac{1}{2}[(x^2 + 4x - 9)^2(2x + 4)x^{-2} + ((x^2 + 4x - 9)^3)(-2)x^{-3}]quad.$$

I hope that you don not expect me to simplify!