A Note on an American Mathematical Monthly “Gem”

In the Feb. 2011 issue of the American Mathematical Monthly, pp. 175-177, there is a “probabilistic” proof (by G. Chang and C. Xu) of the identity \[ \sum_{i=0}^{n} \binom{2i}{i} \binom{2n-2i}{n-i} = 4^n, \] and of a “generalization”. My dear editors (and referees) of the AMM, have you ever heard of the Binomial theorem? Just extract the coefficient of \( x^n \) in \((1 - 4x)^{-1/2}\)^2 = (1 - 4x)^{-1}, and as for the “generalization” do likewise to \((1 - 4x)^{-1/2}\)^m = (1 - 4x)^{-m/2} \(\square\).

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