Math Bite: Constructing Efficient Particle Accelerators is as Easy as 1+1=2 (Thanks to Vladimir Visnjic)

The difficulty in "applied math" is usually not the "math" part (it is often trivial), but the "applied" part, i.e. *realizing* that math can help, and separating the mathematical wheat from the scientific chaff.

A dramatic illustration of the above is Vladimir Visnjic's ingenious construction of two local waves on the particle beam in an accelerator that promises to save the American taxpayer millions of dollars (but, most unfairly, will not earn its discoverer even a penny).

Sources at positions $x = x_1, x_2, \ldots, x_n$ with amplitudes $A_1, A_2, \ldots, A_n$, produce waves of two types, $A_i \sin(x - x_i)$ and $f(x_i)A_i \sin((x - x_i)/2)$. Here $f(x)$ is a certain function (specific to the accelerator) of which we only need to know that it is nonnegative and has zeros. The number of sources, their positions, and amplitudes can be chosen freely up to the constraint $\sum A_i = 0$. The objective is to find the minimal set of sources which creates local waves of both types at the same position.

Visnjic's construction hinges on the following immediate corollary of the celebrated identities $1 + 1 - 2 = 0$ and $1 - 1 + 0 = 0$.

PROPOSITION. *Let $H(x)$ be the Heaviside function (i.e., $H(x) = 1$ if $x > 0$, $H(x) = 0$ otherwise). Then*

$$H(x)\sin(x) + H(x - 2\pi)\sin(x - 2\pi) - 2H(x - 4\pi)\sin(x - 4\pi)$$

vanishes outside $0 < x < 4\pi$. If $f(0) = f(2\pi)$ and $f(4\pi) = 0$, then

$$f(0)H(x)\sin(x/2) + f(2\pi)H(x - 2\pi)\sin(x/2 - \pi) - 2f(4\pi)H(x - 4\pi)\sin(x/2 - 2\pi)$$

vanishes outside $0 < x < 2\pi$.

REFERENCE

1. Vladimir Visnjic, *A Novel Focusing Element for Accelerators*, Phys. Rev. Letters 21 (1994), 2860.

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