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Professor Doron Zeilberger  
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Dear Doron,

Lemma 1 of your paper with Gillis in *Eur. J. Combinatorics* (1983) is a special case of a bijection I hadn't seen before. I'll try to paraphrase here the general idea:

Let  $L$  and  $H$  be disjoint sets of letters (low and high). Given a word  $\alpha \in (L \cup H)^*$  containing  $l$  letters of  $L$  and  $h$  of  $H$ , map it into two words  $(\alpha_L, \alpha_H)$  as follows:  $\alpha_L$  is  $\alpha$  with all elements of  $H$  replaced by  $\square$ , then all blanks in the last  $h$  positions are erased;  $\alpha_H$  is  $\alpha$  with all elements of  $L$  replaced by  $\square$ , then all blanks in the first  $l$  positions are erased. It follows that  $\alpha_L$  and  $\alpha_H$  contain the same number of blanks, say  $k$ ; also the last  $k$  letters of  $\alpha_L$  and the first  $k$  of  $\alpha_H$  are nonblank. Conversely, given any words  $\alpha_L \in (L \cup \{\square\})^*$  and  $\alpha_H \in (H \cup \{\square\})^*$  having these properties, we can uniquely reconstruct  $\alpha$  by filling in the blanks.

Was this construction original with you and Gillis, or "well known" at the time? I want to assign proper credit.

Cordially,

A handwritten signature in cursive script, appearing to be "DK".

Donald E. Knuth  
Professor

DEK/pw

P.S. After writing the above, I seem to have found a generalization of Lemma 2 also. Please see the next page.

P.P.S. Do you know Gillis's full name? He seems to have disappeared from *Math Reviews* after 1984; there are many J. Gillises in the world and I hope to identify him more precisely in the index to my book.

P.P.P.S. Did I mention to you that the 'J. C. P. Miller recurrence' appears in both of Euler's calculus books (1748 and 1755), and is featured quite prominently in the latter?