Professor Doron Zeilberger  
Department of Mathematics  
Temple University  
Philadelphia, PA 19122

Dear Doron,

Lemma 1 of your paper with Gillis in Eur. J. Combinatorics (1983) is a special case of a bijection I hadn't seen before. I'll try to paraphrase here the general idea:

Let $L$ and $H$ be disjoint sets of letters (low and high). Given a word $\alpha \in (L \cup H)^*$ containing $l$ letters of $L$ and $h$ of $H$, map it into two words $(\alpha_L, \alpha_H)$ as follows: $\alpha_L$ is $\alpha$ with all elements of $H$ replaced by $\_\_\_$, then all blanks in the last $h$ positions are erased; $\alpha_H$ is $\alpha$ with all elements of $L$ replaced by $\_\_\_$, then all blanks in the first $l$ positions are erased. It follows that $\alpha_L$ and $\alpha_H$ contain the same number of blanks, say $k$; also the last $k$ letters of $\alpha_L$ and the first $k$ of $\alpha_H$ are nonblank. Conversely, given any words $\alpha_L \in (L \cup \{\_\_\_\})^*$ and $\alpha_H \in (H \cup \{\_\_\_\})^*$ having these properties, we can uniquely reconstruct $\alpha$ by filling in the blanks.

Was this construction original with you and Gillis, or "well known" at the time? I want to assign proper credit.

Cordially,

Donald E. Knuth  
Professor

DEK/pw

P.S. After writing the above, I seem to have found a generalization of Lemma 2 also. Please see the next page.

P.P.S. Do you know Gillis's full name? He seems to have disappeared from Math Reviews after 1984; there are many J. Gillises in the world and I hope to identify him more precisely in the index to my book.

P.P.P.S. Did I mention to you that the 'J. C. P. Miller recurrence' appears in both of Euler's calculus books (1748 and 1755), and is featured quite prominently in the latter?