NAME: (print!) _____

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MATH 436 (Spring 2017), Dr. Z. , Exam 0, Thurs., Jan. 19, 2017 (SEC 211), 11:00-11:40am

1. Write seventeen in (a) base 2 (b) base 3

Ans.(a): seventeen is expressed as: (in base 2)

Ans.(b): seventeen is expressed as: (in base 3)

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2 Prove that $\sqrt{2}$ is irrational.

3 State and prove the Pythagorean Theorem.

4. I am a positive integer. If you divide me by 3, you get remainder 2. If you divide me by 7, you get remainder 6. I am as small as can be (under the above conditions). Who am I?

5. State an explicit expression for

$$\sum_{i=1}^{n} i^2$$

6. State an explicit expression for

$$\sum_{i=1}^{n} i^3$$

7. The sequence of Fibonacci numbers, F_n, are defined by $F_0 = 0$, $F_1 = 1$, and for $n \geq 2$, by

$$F_n = F_{n-1} + F_{n-2} \quad .$$

What is F_7?

8. With F_n defined as above, can you conjecture a nice expression for A_n defined by

$$A_n := F_{n+1}F_{n-1} - F_n^2 \quad .$$

9 What are the usual names for the following functions, given in terms of their power series expansions

$$\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!} \quad ,$$

$$\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} \quad ,$$

$$\sum_{n=0}^{\infty} x^n \quad .$$

10. Order the following famous mathematicians according to their year of birth, from oldest (most ancient) to the youngest (most modern).
Gauss, Archimedes, Zeilberger, Euler, Hilbert, Laplace, Galois, Cayley .
For each, state the century of birth.

11. What does Fermat's Last Theorem Claim? Who proved it?

12. What is the Riemann Zeta Function, $\zeta(s)$. What Is the Riemann Hypothesis?

13. List the first 10 prime numbers.

14. Prove that there are infinitely many primes.

15. The perfect Platonic solids are the tetrahedron (four faces), cube (six faces), octahedron (eight faces), dodecahedron (12 faces), and icosahedron (20 faces). Do you know of

a relation between the number of vertices, V, the number of edges, E, and the number of faces, F? (For example, for a cube $V = 8$, $E = 12$, $F = 6$. Who discovered this relation?

16. Is it possible to construct a square with the same area of a given circle, only using straight-edge and compass?

17. If you toss a fair coin 10 times, what is the probability of getting at most two Heads?