Quiz 2
Wednesday, February 8

1. (2 pts) Determine the interval where there is certainly a unique solution to the IVP

\[ \begin{align*}
\text{(ln} t) y' + y &= \frac{1}{t^3}, \\
y(2) &= 3.
\end{align*} \]

Rewrite in standard form:

\[ y' + \frac{1}{\ln t} y = \frac{1}{(t-3)(\ln t)}, \quad \text{P(t) discontinuous at t=3} \]
\[ q(t) \text{ decreses at t=1.3, need an interval containing t=2.} \]

2. (3 pts) Find the equilibrium of the autonomous equation

\[ y' = y^2 - 5y + 6 \]

and determine the stability.

\[ y' = (y-3)(y-2). \]

Sketch of sol'n:

Phase line:

3. (5 pts) Solve the IVP on \( x > 0 \).

\[ \begin{align*}
\frac{y}{x} + 6x + (\ln x - 2)y' &= 0 \\
y(1) &= 0.
\end{align*} \]

\( M = \frac{y}{x} + 6x, \quad N = \ln x - 2. \)

We check:

\[ My = \frac{1}{x}, \quad Nx = \frac{1}{x}. \]

So the equation is exact.

Now need to find \( \psi \) such that

\[ \begin{align*}
\psi_x &= M = \frac{y}{x} + 6x, \\
\psi_y &= N = \ln x - 2.
\end{align*} \]

\[ \psi(x,y) = \int M \, dx = \int \frac{y}{x} + 6x \, dx = y \ln x + 3x^2 + f(y). \]

(since \( x > 0 \)) = \( y \ln x + 3x^2 + f(y) \).

Differentiate w.r.t. to \( y \), and compare with \( N \)

\[ y_\gamma = (\ln x + f'(y)) = \ln x - 2 \Rightarrow f'(y) = -2 \]
\[ f(y) = -2y + C. \]

So \( \psi(x,y) = y \ln x + 3x^2 - 2y + C \)

\( \psi(x,y) \) is given by \( y \ln x + 3x^2 - 2y + C \) 

Plug in \( y(1) = 0 \): \( 0 = \frac{C - 3}{\ln 1 - 2} \Rightarrow C = 3 \), solution is \( y = \frac{3 - 3x^2}{\ln x - 2} \).