Math 151, Midterm 2 – Review

Do not assume that the second midterm will be similar to this review sheet. Your second exam may contain questions that do not resemble any of the questions on this review.

Textbook Problems

Do not assume that merely completing all WebAssign work is sufficient preparation for the midterms. In particular, you will need to show all necessary steps on the exam, not just give an answer. It is strongly recommended that you work out all textbook problems listed on the department’s Math 151 website.

Non-Textbook Problems

1. Find the derivative of the functions.
   a) \( f(x) = 10^{\arccos x} \)
   b) \( g(x) = \sin^{-1} \left( \log_5 \left( x^3 + x + 1 \right) \right) \)
   c) \( h(x) = \ln \left( \arctan x \right) \)

2. Evaluate the limits.
   a) \( \lim_{x \to 1} \frac{2x^4 - 3x^3 + x^2 - x + 1}{x^4 - 3x^3 + 2x^2 + x - 1} \)
   b) \( \lim_{x \to 0} \frac{x - \sin x}{x - x \cos x} \)
   c) \( \lim_{x \to 0^+} \left( x^{1/10} \ln x \right) \)
   d) \( \lim_{x \to \infty} \frac{\ln x}{x^{1/10}} \)

3. Find the horizontal asymptotes of \( f(x) = \frac{x}{\sqrt{7x^2 + 1}} \).

4. For each function given below, find the domain, \( x- \) and \( y- \)intercepts, the intervals where it is increasing, the intervals where it is decreasing, the intervals where it is concave up, the intervals where it is concave down, the local maxima and minima, and the inflection points. Sketch the graph of the function.
   a) \( f(x) = x^5 - 3x^3 + 4x \)
   b) \( g(x) = x^2 - \frac{1}{x} \)
   c) \( k(x) = \frac{1}{\sqrt{x^2 + 1}} \)
   d) \( m(x) = \cos x - \frac{\sin x}{\sqrt{3}} \) where \( x \in [-2\pi, 2\pi] \)
   e) \( h(x) = x - \tan(x) \) where \( x \in (-\pi/2, \pi/2) \)

5. Let \( f(x) = \sec^{-1} x \). Find \( f'(x) \) using the identity \( \sec^{-1} x = \cos^{-1} \left( 1/x \right) \).

6. Find the absolute extrema of the function \( f(x) = 8x^5 - 5x^4 \) on the interval \([-1, 1]\).

7. Find the absolute maximum and absolute minimum of the function.
   \[
   f(x) = \begin{cases} 
   9 - 4x, & \text{if } 0 \leq x < 1 \\
   -x^2 + 6x, & \text{if } 1 \leq x \leq 4 
   \end{cases}
   \]

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8. Find the derivative of \( f(x) = [(\ln x)^3] \) for \( x > 1 \).

9. Find the equation of the tangent line to graph of \( x^2y^3 + xy = 79 \) at the point \((3, 2)\).

10. Use linear approximation to estimate \( 26^{1/3} \). Is your answer an overestimate or an underestimate? Explain.

11. A spherical helium balloon is given a very small amount of extra helium, so that its radius increases by 0.001 percent. What is the percentage increase in its volume? What is the percentage increase in its surface area?

12. A spherical weather balloon is being inflated at the rate of 12 cubic inches per second. What is the radius of the balloon when its surface area is increasing at a rate of 5 square inches per second?

13. Find the area between the \( x \)-axis, the line \( x = 1 \) and the parabola \( y = x^2 \) in the following way: Approximate the area using the sum of the areas of \( n \) rectangles. Let \( n \) approach infinity.

14. Find the first two successive approximations when Newton’s method with initial guess 2 is used to approximate \( \sqrt{2} \).

15. Find all functions \( f(x) \) such that \( f''(x) = x(x^2 - 1) \).

16. Consider the function \( f(x) = x^{2/3}(1 - x^2)^2 \). Find the intervals where it is increasing, the intervals where it is decreasing, the intervals where it is concave up, the intervals where it is concave down, the local maxima, the local minima and the inflection points.

17. Consider the function \( f(x) = \frac{x}{1 + x^2} \). Find the intervals where it is increasing, the intervals where it is decreasing, the intervals where it is concave up, the intervals where it is concave down, the absolute maxima, the absolute minima and the inflection points.

18. A closed box with rectangular sides is built according to the following specifications: The top and bottom sides are made of a material that costs $5 per square foot, and the four vertical sides are made of a material that costs $3 per square foot. The top and bottom sides are rectangles with sides of length \( x \) and \( y \), where \( 2x = 7y \). The total cost of materials is $100. Find the largest possible volume that such a box can hold.

19. We want to make a conical drinking cup out of paper. It should hold exactly 100 cubic inches of water. Find the dimensions of a cup of this type that minimizes the surface area of the cup.

20. We want to build a cylindrical can with total surface area of 100 square inches. This surface area includes the top and bottom of the can. Find the dimensions of the can of this type that maximizes the volume of the can.

21. Let \( f(x) \) be a differentiable function such that \( f(4) = 5 \) and \( f'(x) < -2 \) for all \( x \). What can we conclude about \( f(7) \)?

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