Math 151, Midterm 2 – Answers to some Review Problems

1. a) \[ f'(x) = -\frac{(\ln 10) 10 \arccos x}{\sqrt{1 - x^2}} \]
   b) \[ g'(x) = \frac{3x^2 + 1}{(\ln 5)(x^3 + x + 1)\sqrt{1 - \left[\log_5 (x^3 + x + 1)\right]^2}} \]
   c) \[ h'(x) = \frac{1}{(1 + x^2) \arctan x} \]

2. a) \(-4\)
   b) \(1/3\)
   c) \(0\)
   d) \(0\)

3. The horizontal asymptotes are \(y = \frac{1}{\sqrt{7}}\) and \(y = -\frac{1}{\sqrt{7}}\).

4. a) \(f(x) = x^5 - 3x^3 + 4x\)

   The function is increasing on \((-\infty, -1)\), \((-2/\sqrt{5}, 2/\sqrt{5})\), and \((1, \infty)\). The function is decreasing on \((-1, -2/\sqrt{5})\), and \((2/\sqrt{5}, 1)\). The local maxima occur at \(x = -1\) and \(x = 2/\sqrt{5}\). The local minimum occur at \(x = -2/\sqrt{5}\), and \(x = 1\). The function is concave up on \((-3/\sqrt{10}, 0)\), and \((3/\sqrt{10}, \infty)\). The function is concave down on \((-\infty, -3/\sqrt{10})\) and \((0, 3/\sqrt{10})\). The inflection points are at \(x = 0\) and \(x = \pm 3/\sqrt{10}\).

b) \(g(x) = x^2 - \frac{1}{x}\)

   The critical point occurs at \(x = -1/\sqrt{2}\). There is an inflection point at \(x = 1\).

c) \(g(x) = \frac{1}{\sqrt{x^2 + 1}}\)

   The inflection points occur at \(x = \pm 1/\sqrt{2}\). There is a critical point at \(x = 0\).

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d) \( m(x) = \cos x - \frac{\sin x}{\sqrt{3}} \) where \( x \in [-2\pi, 2\pi] \).

The critical point occur at \( x = \frac{-7\pi}{6}, \frac{-\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6} \).

There are inflection points at \( x = \frac{-5\pi}{3}, \frac{-2\pi}{3}, \frac{\pi}{3}, \frac{4\pi}{3} \).

e) \( h(x) = x - \tan(x) \) where \( x \in (-\pi/2, \pi/2) \)

There is a critical point at \( x = 0 \) and there is an inflection point at \( x = 0 \).

5. ...

6. The absolute maximum is at \( x = -1 \) and the absolute minimum is at \( x = -\sqrt{2} \).

7. Find the absolute maximum is 9 and the absolute minimum is 5.

8. \( f'(x) = (\ln x)^x \left( \ln (\ln x) + \frac{1}{\ln x} \right) \)

9. \( y - 2 = -\frac{50}{111} (x - 3) \).

10. \( 26^{1/3} \approx \frac{80}{27} \) It is an overestimate.

11. The volume increases by approximately 0.003% and the surface area increases by approximately 0.002%.

12. The radius is 24/5 inches.

13. The total area of \( n \) rectangles is \( \frac{1}{6} \cdot \frac{n+1}{n} \cdot \frac{2n+1}{n} \). This approaches the exact area of 1/3 as \( n \) approaches infinity.

14. If \( x_0 = 2 \) then \( x_1 = 3/2 \) and \( x_2 = 17/12 \).

15. \( f(x) = \frac{1}{20} x^5 - \frac{1}{6} x^3 + Cx + d \) for any real numbers \( C \) and \( D \).

16. The function is increasing on \((-1, -1/\sqrt{7}), (0, 1/\sqrt{7}), \) and \((1, \infty)\). The function is decreasing on \((-\infty, -1), (-1/\sqrt{7}, 0) \) and \((1/\sqrt{7}, 1)\). The local maxima occur at \( x = \pm -1/\sqrt{7} \). The local minima occur at \( x = \pm 1 \) and \( x = 0 \).

The inflection points occur at
\[
x = \pm \sqrt{\frac{20 + \sqrt{477}}{77}}
\]
The function is concave up on
\[ (-\infty, -\sqrt{\frac{20 + \sqrt{477}}{77}}) \quad \text{and} \quad \left( \sqrt{\frac{20 + \sqrt{477}}{77}}, \infty \right) \]

The function is concave down on
\[ \left( -\sqrt{\frac{20 + \sqrt{477}}{77}}, 0 \right) \quad \text{and} \quad \left( 0, \sqrt{\frac{20 + \sqrt{477}}{77}} \right) \]

17. The function is increasing on \((-1, 1)\). The function is decreasing on \((-\infty, -1)\) and \((1, \infty)\). The function is concave up in \((-\sqrt{3}, 0)\) and \((\sqrt{3}, \infty)\) and teh function is concave down on \((-\infty, -\sqrt{3})\) and \(0, \sqrt{3}\). There is an absolute maximum at \(x = 1\). There is an absolute maximum at \(x = -1\). There are inflection points at \(x = 0\) and \(x = \pm \sqrt{3}\).

18. The volume is maximized when \(x = \sqrt{\frac{35}{3}}\) feet, \(y = \frac{2\sqrt{35}}{7\sqrt{3}}\) feet, and the height of the box is \(\frac{20\sqrt{35}}{27\sqrt{3}}\) feet.

19. The surface area is minimized when the height of the cone is \(\left( \frac{600}{\pi} \right)^{1/3}\).

20. The volume is maximized when the radius \(R\) is \(\sqrt{\frac{50}{3\pi}}\) inches and the height \(H = 2R\).

21. Using the Mean Value Theorem, we can conclude that \(f(7) < -1\).