Practice 2 for Feb 7

Consider the vector 
\[ \mathbf{v} = (-4, 2, 4) \]

a) Find a unit vector \( \mathbf{w} \) in the direction of \( \mathbf{v} \)

\[ \mathbf{w} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{(-4, 2, 4)}{\sqrt{16+4+16}} = \left< \frac{-4}{\sqrt{36}}, \frac{2}{\sqrt{36}}, \frac{4}{\sqrt{36}} \right> \]

b) Suppose \( \mathbf{u} = (x, 1, 2) \) is another vector and 
\[ \text{proj } \mathbf{u} \mathbf{v} = \frac{3}{36} \mathbf{v} \]

Find \( x \)

\[ \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|^2} = \frac{3}{36} \mathbf{v} \]

\[ \frac{-4x + 4 - 4}{36} \mathbf{v} = \frac{3}{36} \mathbf{v} \]

\[ \Rightarrow -4x = 3 \]

\[ \Rightarrow x = \frac{-3}{4} \]

C) What is the value of \( x \) if 
\[ \text{proj } \mathbf{v} \mathbf{w} = \frac{3 \mathbf{w}}{36} \]

\[ \text{proj } \mathbf{w} \mathbf{v} = \text{proj } \mathbf{w} \mathbf{w} \]

so the same value of \( x \) works.
(a) Find the point of intersection between the lines
\[ x = 2t, \ y = t + 2, \ z = 3t - 1 \]
\[ x = 5k - 2, \ y = k + 4, \ z = 5k + 1 \]

\[ \begin{align*}
2t &= 5k - 2 \quad \rightarrow \quad 2t = 5t - 10 - 2 \\
t + 2 &= k + 4 \quad \rightarrow \quad k = t - 2 \\
3t - 1 &= 5k + 1 \\
k &= 2
\end{align*} \]

Point \( P = (8, 6, 11) \)

(b) Find the equation of the plane which contains both lines.

\[ n^o \times (2, 1, 3) \times (5, 1, 5) \]
\[ = \det \begin{pmatrix}
2 & 3 \\
5 & 5
\end{pmatrix}
\]
\[ = (2, 5, -3) \]

\[ 2x + 5y - 3z = (2, 5, -3) \cdot (8, 6, 11) = 13 \]
The planes with equations

\[ \begin{align*}
5x - 6y - 2z &= 0 \\
2x + 3y - 2z &= -7
\end{align*} \]

intersect on a line.

A. Suppose that the point \( P,(0, y, z) \) belongs to this line. Then

\((0, y, z)\) satisfies the equations

\[ \begin{cases} 
5y - 2z = 0 \\
2y - 2z = -7
\end{cases} \]

so

\[ 11Y = -13 \]

\[ Y = -\frac{13}{11} \]

Substitute in (2)

\[ -\frac{65}{11} - 2z = -7 \]

\[ -\frac{65}{11} + 7 = 2z \]

\[ \frac{12}{2} = 2 \]

\[ z = 6 \]

Suppose \( v^2 = (v_1, v_2, v_3) \) is a tangent vector to this line. Then

\[ \frac{v_2}{v_1} = 2, \quad \frac{v_3}{v_1} = 2 \]

\[ \bar{v}^2 = (2, 5, -2) \]
\[ \mathbf{V}^0 = \mathbf{v_1} \times \mathbf{v_2} \]
\[ \mathbf{V}^0 = \det \begin{pmatrix} \mathbf{I}^2 & \mathbf{J}^3 & \mathbf{K}^3 \\ \mathbf{E} & -6 & -2 \\ 2 & 5 & -2 \end{pmatrix} \]
\[ \mathbf{V}^0 = (22, 6, 371) = (v_1, v_2, v_3) \]

\[ \frac{v_2}{v_1} = \frac{6}{22} \]
\[ \frac{v_3}{v_1} = \frac{371}{22} \]
Consider the planes with equations

\[ 12x + 7y - 8z = 87 \]
\[ 4x + 9y - 4z = 27 \]

If \( \langle 1, 4, C \rangle \) is parallel to the line of intersection of the two planes, find \( C \).

The direction of the line is given by

\[ \vec{n}_1 \times \vec{n}_2 = \langle 24, 16, 40 \rangle \]
\[ = 4 \langle 1, 4, 10 \rangle \]

Thus

\[ C = 10 \]

Suppose \( (A, B, 3) \) belongs to this line.

Then the values of \( A \) and \( B \) are

plug into equations of the plane

\[ \begin{cases} 12A + 51 - 8B = 87 \\ 4A + 27 - 4B = 27 \end{cases} \]

\[ \Rightarrow \begin{cases} 12A - 8B = 36 \\ 4A - 4B = 0 \Rightarrow |A| = |B| \end{cases} \]

in the first equation we get \( 4A = 36 \)

\[ \begin{align*}
A &= 9 \\
B &= 9
\end{align*} \]