This examination booklet contains 6 questions on 10 pages of paper including the front cover.

Do all of your work in this booklet, show all your computations and justify/explain your answers. Your justification must be based on techniques already discussed in this course. If asked to evaluate an integral, remember to show all the steps that gives you its value.

Except for your personal note sheet, no other resources like class notes, books, calculator, etc are allowed. Remember that your note sheet must be handwritten, on both sides of a single sheet of paper.

Unless otherwise state, give exact answers. For example, write $\pi$ and $\sqrt{2}$ instead of 3.14 and 1.41. However, when an expression simplifies to a well-known value, you must use that value. For example, you must write 1 instead of $e^0$, and you must write $\pi/3$ instead of $\sec^{-1}(2)$.

If you run out of space when answering a problem, you may use any of the last three pages of the exam, but you must: indicate in the space below the question that you are continuing your answer on the extra sheet, and indicate on the extra sheet which problem you are working on.

Do not discuss the exam with anyone until grades are posted on Canvas.

WRITE OUT AND SIGN PLEDGE [1 point]

On my honor, I have neither received nor given any unauthorized assistance on this examination.
Problem 1. [16 points] a) Find the vector equation $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$ which solves the system

\[
\begin{align*}
\frac{d\mathbf{r}}{dt} &= \langle -10 \cos t, 9 \sin t, \sqrt{19} \sin t \rangle \\
\mathbf{r}(0) &= \langle 0, -9, -\sqrt{19} \rangle
\end{align*}
\]

b) Find the arc-length parameter $s(t)$ as a function of $t$, and use this to find the arc-length parametrization of the curve traced out by $\mathbf{r}(t)$. 
Problem 2. [15 points] Consider the curve with equation
\[ r(t) = (t^2 - 3t)i + (5t^2 - 2)j \]
Find the values of \( t \) where the velocity to this curve is parallel to the vector \( 27i + 5j \).
Problem 3. [20 points] Consider the function

\[ f(x, y) = \frac{x + y}{\sqrt{x^2 + y^2}} \]

a) Find the domain of the function.

b) Find \( \lim_{(x,y) \to (0,0)} f(x, y) \) or demonstrate that the limit does not exist.

c) Write the equation(s) of the level curve \( f(x, y) = 1 \) and draw it on the \( xy \) plane.
Problem 4. [20 points] Find all the points on the ellipsoid
\[
\frac{x^2}{64} + \frac{y^2}{25} + z^2 = 1
\]
where the tangent plane is normal to \( \mathbf{n} = \langle 1, 1, -2 \rangle \).
Problem 5. [16 points] Consider the function

\[ F(x, y, z) = 2xz + 7y - z^2 \]

a) What is the equation of the level surface that passes through the point \( P = (1, 7, 0) \)?

b) At the same time, the previous equation defines \( z \) implicitly as a function of \( x \) and \( y \). What are the previous values of \( \frac{\partial z}{\partial x} \) and \( \frac{\partial z}{\partial y} \) at the point \( P \)?
Problem 6. [12 points] Consider

\[ f(x, y) = \frac{1}{2} (e^{ay} - e^{-5y}) \sin(5x) \]

Find the values of \( a \) that make \( f \) satisfy the equation \( \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0 \).
YOU CAN USE THIS PAGE FOR ANY QUESTION OF THE EXAM OR SCRATCH-WORK: