This examination booklet contains 6 questions on 10 pages of paper including the front cover.

Do all of your work in this booklet, show all your computations and justify/explain your answers. Your justification must be based on techniques already discussed in this course. If asked to evaluate an integral, remember to show all the steps that gives you its value.

Except for your personal note sheet, no other resources like class notes, books, calculator, etc are allowed. Remember that your note sheet must be handwritten, on both sides of a single sheet of paper.

Unless otherwise state, give exact answers. For example, write $\pi$ and $\sqrt{2}$ instead of 3.14 and 1.41. However, when an expression simplifies to a well-known value, you must use that value. For example, you must write 1 instead of $e^0$, and you must write $\pi/3$ instead of sec$^{-1}(2)$.

If you run out of space when answering a problem, you may use any of the last three pages of the exam, but you must: indicate in the space below the question that you are continuing your answer on the extra sheet, and indicate on the extra sheet which problem you are working on.

Do not discuss the exam with anyone until grades are posted on Canvas.

WRITE OUT AND SIGN PLEDGE

On my honor, I have neither received nor given any unauthorized assistance on this examination.
Problem 1. [∗] Sketch the region of integration of
\[ \int_0^1 \int_1^{e^x} 10 dy \, dx \]
and write a double integral with the order of integration reversed. Do not compute it.
Problem 2. [*] Change

\[ \int_0^{12} \int_0^{\sqrt{36-(x-6)^2}} \frac{x + y}{x^2 + y^2} dy dx \]

into an equivalent polar integral. Then evaluate the polar integral.
Problem 3. [*] Consider the function \( f(x, y, z) = x^2 + y^2 \), and let \( D \) be the solid region outside the cylinder \( x^2 + y^2 = 16 \) and inside the sphere \( x^2 + y^2 + z^2 = 64 \). Express
\[
\iiint_D f(x, y, z) \, dV
\]
as a triple integral in spherical coordinates. Do not compute it.
Problem 4. [*] Let $D$ be the region bounded below by the plane $z = 0$, above by the sphere $x^2 + y^2 + z^2 = 4$, and on the sides by the cylinder $x^2 + y^2 = 1$. Set up a triple integral for the volume of $D$ using the order $dzdrd\theta$ from cylindrical coordinates. Do not compute it.
Problem 5. [*] A space probe in the shape of the ellipsoid
\[ x^2 + 4y^2 + 4z^2 = 16 \]
enters Earth’s atmosphere and the surface begins to heat. After 1 hour, the temperature at the point \((x, y, z)\) on the probe’s surface is \(T(x, y, z) = 4yz\).

Suppose you want to find the hottest point (or points) on the probe’s surface. Use the method of Lagrange multipliers to find such points.
Problem 6. [*] Consider the double integral

$$\int \int_R (y - x) \, dA$$

where $R$ is the parallelogram joining $(1, 2), (3, 4), (4, 3)$ and $(6, 5)$. Find the value of this integral using the change of variables $u = x - y$ and $v = x - 3y$. 
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