This examination booklet contains 6 questions on 10 pages of paper including the front cover.

Do all of your work in this booklet, show all your computations and justify/explain your answers. Your justification must be based on techniques already discussed in this course. If asked to evaluate an integral, remember to show all the steps that gives you its value.

Except for your personal note sheet, no other resources like class notes, books, calculator, etc are allowed. Remember that your note sheet must be handwritten, on both sides of a single sheet of paper.

Unless otherwise state, give exact answers. For example, write \( \pi \) and \( \sqrt{2} \) instead of 3.14 and 1.41. However, when an expression simplifies to a well-known value, you must use that value. For example, you must write 1 instead of \( e^0 \), and you must write \( \pi/3 \) instead of \( \sec^{-1}(2) \).

If you run out of space when answering a problem, you may use any of the last three pages of the exam, but you must: indicate in the space below the question that you are continuing your answer on the extra sheet, and indicate on the extra sheet which problem you are working on.

Do not discuss the exam with anyone until grades are posted on Canvas.

WRITE OUT AND SIGN PLEDGE

On my honor, I have neither received nor given any unauthorized assistance on this examination.
Problem 1. [*] Consider the region between the paraboloid \( z = \frac{x^2}{2} + y^2 - 49 \) and the parabolic cylinder \( z = 49 - \frac{x^2}{2} \). Write but do not compute, the volume in \textbf{cartesian coordinates} as a triple integral.
Problem 2. [*] Write but do not compute a triple integral in spherical coordinates for the volume inside the sphere $x^2 + y^2 + (z - 6)^2 = 36$ and outside the sphere $x^2 + y^2 + z^2 = 36$. 
Problem 3. Consider the region $D$ in the first quadrant determined by the curves $x = 5$, $x = \ln \left( \frac{y}{7} \right)$, $x = 0$, $y = 0$. Find

$$\iint_D \ln(1 + e^x) \, dA$$
Problem 4. [*] Find the integral of

\[ f(x, y) = \frac{y^3}{(x^2 + y^2)^2} \]

over the region in the first quadrant inside the circle \((x - 9)^2 + y^2 = 81\) and inside the triangle with vertices \((0, 0), (18, 18)\) and \((18, 0)\).
Problem 5. [*] Suppose a box is located in the first octant with a vertex at the origin and its sides are parallel to the coordinate axes and one of the vertices belonging to the surface with equation $7 = x^2 + y^2 + 4z$.

Use the Lagrange Multiplier method to find the coordinates $(x, y, z)$ of the vertex on the previous surface which yields the maximum volume for the box.
Problem 6. [*] Consider the region $R$ determined by the inequalities

$$0 \leq x \leq 5, \quad -\frac{x}{5} \leq y \leq 4 - x$$

Use the change of variables

$$x = 5u, \quad y = 2v - u$$

to write (but do not integrate) the double integral

$$\int\int_{R} x^2 \sqrt{x + 5y} \, dA$$

in terms of the variables $u$ and $v$. 

YOU CAN USE THIS PAGE FOR ANY QUESTION OF THE EXAM OR SCRATCH-WORK:
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