

Oral Qualifying Exam

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1 Combinatorics

Set Systems

- **Basic Enumeration:** combinatorial and bijective proofs, generating functions, binomial and multinomial coefficients, Inclusion-Exclusion, Stirling's formula, recurrence relations, Catalan numbers.
- **Posets and Hypergraphs:** Dilworth's theorem, Sperner's theorem, LYM inequality, Erdős-Ko-Rado theorem, Kruskal-Katona theorem, Fisher's inequality, Ray-Chaudhuri-Wilson theorem, Frankl-Wilson theorem, Baranyai's theorem
- **Ramsey Theory:** Ramsey's theorem for graphs and hypergraphs, infinite Ramsey, upper bounds, probabilistic lower bounds, van der Waerden's theorem, statement of Hales-Jewett theorem, Schur's theorem, Rado's theorem, Erdős-Szekeres ("happy ending") theorem

Probabilistic Methods

- **Basics:** linearity of expectation, union bound, Markov's inequality, Chernoff bounds
- **Alterations:** lower bound for diagonal Ramsey numbers, graph with high girth and high chromatic number
- **Second Moment Method:** Chebyshev's inequality, threshold function to contain a fixed subgraph, Rödl Nibble
- **Lovász Local Lemma:** statement of symmetric and general versions, application to lower bounds for diagonal Ramsey numbers
- **Correlation Inequalities:** Ahlswede-Daykin/Four Functions Theorem, Harris-Kleitman, FKG inequality for sets, Janson inequalities
- **Martingales:** definition, vertex and edge exposures, Azuma's inequality, application to chromatic number, Talagrand's inequality

Graph Theory

- **Matchings:** König's Theorem, Hall's Marriage Theorem, Tutte's Theorem
- **Connectivity:** Kruskal and Prim spanning tree algorithms, Menger's Theorem, max-flow min-cut
- **Planarity:** Euler's formula, Kuratowski's theorem
- **Coloring:** Five-Color Theorem, Brooks' theorem, Vizing's theorem, Thomassen's Theorem on list-coloring of planar graphs
- **Extremal:** Mantel's Theorem, Turán's Theorem, Erdős-Stone-Simonovits Theorem, statement of Szemerédi's Regularity Lemma

2 Combinatorial Group Theory

- **Group Presentations:** definition and elementary properties, Dehn's fundamental problems and correspondence to fundamental group
- **Free Groups:** universal property, rank of a free group, characterization via freely reduced words and application to the word problem, cyclically reduced words, orders of elements, Cayley graphs, free actions on trees
- **Subgroups of Free Groups:** Nielsen-Schreier Subgroup Theorem, Schreier graph, properties of subgroups of finite index, construction of a basis for subgroups of F_2
- **Free Products:** definition and properties, Normal Form Theorem, words of finite order, Kurosh Subgroup Theorem
- **Free Products with Amalgamation:** definition and properties, Normal Form Theorem, words of finite order, $SL_2(\mathbb{Z})$, $PSL_2(\mathbb{Z})$, examples of trees of amalgams