Math 112
Review Exercises for the Final Exam

The following are review exercises for the Math 112 final exam. These exercises are provided for you to practice or test yourself for readiness for the final exam. There are many more problems appearing here than would be on the final. These exercises represent many of the types of problems you would be expected to solve on the final, but are not meant to represent all possible types of problems that could appear on the final exam.

Your final exam will be in two parts: the first part does not allow the use of a calculator, and the second part does allow the use of a graphing calculator. Since the exercises in this review sheet are mixed together, we have put a ▭ symbol next to exercises or parts of exercises where you WILL be allowed to use the graphing calculator: otherwise you should be able to solve the problem WITHOUT a calculator. Such a symbol will not be on the final exam. Please note that for the final, you may use any graphing calculator except the TI-89, TI-Inspire, and any calculator with a QWERTY keypad.

Show all your work: unsupported results may not receive credit.

1. Sketch the graph of the following:

Using interval notation, state the domain and the range. State the equation(s) of the asymptote(s). Find the x- and y-intercepts where they exist.

(a) $f(x) = 3^{x-2} + 4$

(b) $f(x) = \ln(x-2) + 3$

(c) $f(x) = 5^{x-3} + 1$

(d) $f(x) = \log(x-1) + 2$

2. Given $\log_a 5 = 2.3$ and $\log_a 3 = 1.6$, fill in the table below with the appropriate values.

X	15	9	$\frac{5}{3}$	5a	$\frac{3}{a^2}$
$\log_a x$					

3. Which is the following is larger: $\log_3 28$ or $\log_4 63$? JUSTIFY YOUR ANSWER FOR CREDIT

4. Find the EXACT solution for the following:

(a) $\log(x+2) - \log(x) = 3$

(b) $\log(x-3) = 1 - \log(x)$

(c) $2^{2x+1} = 8^{3x}$

(d) $3^{2x-3} = 27^{x+3}$

(e) $x^2 e^x - 2e^x = 0$

5. Given the following equations: Find (i) all solutions in the interval $[0,2\pi)$ in radians, and (ii) all solutions in radians:

(a) $(2\sin x - \sqrt{3})(\sin x - 2) = 0$
(b) $2\sin^2 x + \sin x - 3 = 0$
(c) $\tan^2 x - 1 = 0$

6. Solve for x accurate to 2 places if x is in the interval $(0,\pi)$: $\sin(4x) = \ln(x+2)$.

7. For the functions below, find the period, amplitude, phase shift, and sketch the graph of one period. Be sure to label all x-intercepts and maximum and minimum points.

(a) $f(x) = 6\cos\left(2x - \frac{\pi}{3}\right)$
(b) $f(x) = -5\sin\left(3x + \frac{\pi}{2}\right)$

8. Find the EXACT value of

(a) $\sin(15^o)$
(b) $\cos(75^o)$

9. Find the reference angle for -135^o. Find $\cos(-135^o)$ exactly

10. (a) If $\cos t = \frac{4}{7}$ and $\frac{3\pi}{2} < t < 2\pi$, Find the remaining values of the trigonometric functions of **t.**

(b) If $\sin t = -\frac{3}{8}$ and $\frac{3\pi}{2} < t < 2\pi$, Find the remaining values of the trigonometric functions of **t.**

11. Find the values of the trigonometric functions of θ if the terminal side of θ contains the point $(-4,5)$.

12. Find:

(a) $\cos 2x$ given: $\cos x = -\frac{4}{5}$ and x is in Quadrant II.

(b) $\sin 2x$ given: $\sin x = \frac{2}{5}$ and x is in Quadrant II.

(c) $\sin(x+y)$ given: $\sin x = -\frac{2}{3}$ and x is in Quadrant III; and $\sin y = \frac{1}{3}$ and y is in Quadrant II.

13. Verify the identity:

(a) $\dfrac{1-\sin^2 x}{\cot^2 x} = \sin^2 x$

(b) $\dfrac{\cos x}{1+\sin x} + \dfrac{\cos x}{1-\sin x} = 2\sec x$

14. Find the following exactly in degrees:

(a) $\sin^{-1}\left(-\dfrac{1}{2}\right)$

(b) $\cos^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)$

15. Find the following EXACT:

(a) $\sin[\cos^{-1}(2/3)]$

(b) $\sin[\tan^{-1}(-6)]$

16. If $\theta = \arcsin x$, express $\tan\theta$ in terms of x.

17. (a) Suppose on Jan 1, 1997 Dave invested \$2,000 into a bank account at 5% interest compounded continuously. Let ***y(t)*** be the value of Dave's investment after ***t*** years. Give an exact formula for ***y(t)***
(b) Also on Jan 1, 1997 John decides to invest. He put \$2,500 into an account at 3% interest compounded monthly. Let ***g(t)*** be the value of John's investment after ***t*** years. Give an exact formula for ***g(t)***.
(c) Which account is worth more after 9 years? [Must show work to receive credit.]
(d) To the nearest tenth, at what time ***t*** is the value of both accounts the same?

18. Evaluate the following EXACTLY.

(a) $\log_2 \sqrt[7]{64}$

(b) $\log_3 \sqrt[7]{81}$

19. Evaluate to two decimal places.

(a) $\log_5 27$

(b) $\log_3 15$

20. Given the following functions $f(x)$ below, find $f^{-1}(x)$.

(a) $f(x) = 3^x$

(b) $f(x) = \log_3 x$

(c) $f(x) = 10^{x+1} - 3$

(d) $f(x) = \log(x-4) + 5$

(e) $f(x) = e^{2x+1} - 4$

(f) $f(x) = \ln(2x+3) - 5$

21. Sketch a graph of the following. Label the asymptotes. Find the intercepts EXACT. Find $f^{-1}(x)$. Find the intercepts accurate to two decimal places.

(a) $f(x) = e^{x+2} - 3$

(b) $f(x) = \ln(x+4) + 1$

22. From 1990 to 2000 the student tuition at a University grew from \$12,000 to \$18,000.

(a) Using the exponential growth model, determine ***r***, the annual rate of increase for the population as a decimal accurate to 3 places

(b) Assuming the same growth rate use ***r*** found in Part (a) above, find in what year (to the nearest year) the tuition of Rutgers will reach \$30,000.

23. Carbon dating is commonly used to determine how old an object is by measuring the amount of carbon-14 that is left in an object as the object decays over the years. This decay proceeds exponentially with a half-life of approximately 5800 years. How old (to the nearest year) would carbon dating say a piece of bone is when the amount of carbon-14 has decayed from its original amount of 100 grams to a final amount of 22 grams?

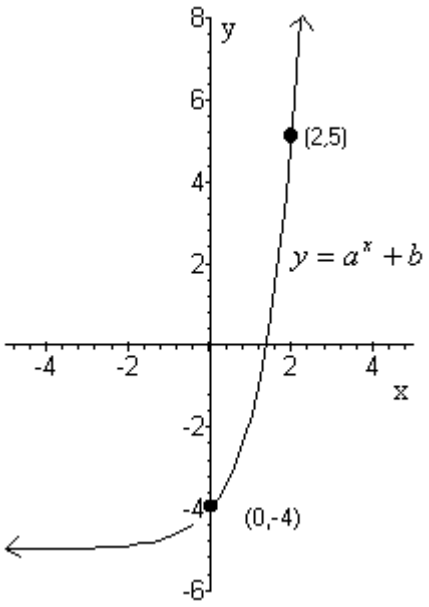
24. Air Temperature *T* varies in a periodic manner, with highs during the day and lows during the night. Assume that the weather pattern is the same for one week, and suppose that *T* (in degrees Fahrenheit) at a particular time of day is given by the function

$$T(t) = 13\sin\left(\frac{\pi}{12}t - \frac{2\pi}{3}\right) + 62$$

where *t* is the time in hours (with *t=0* at midnight). [All answers EXACT.]

(a) What is the highest temperature during the day?
(b) What is the lowest temperature during the day?
(c) Find the temperature at 12 PM.
(d) Find the temperature at 6 PM.
(e) What is the period of *T(t)*?

25. The graph of $y = a^x + b$ is shown below. Find the EXACT values of *a* and *b*.



26. A ramp 17 feet in length rises to a loading platform that is 4 feet off the ground. Find the angle θ that the ramp makes with the ground. (Give your answer accurate to 2 decimal places.)

27. Find the length of a 25^o arc with radius 12 inches accurate to 2 decimal places.

28. Two trains, Train A and Train B, leave a train station at 10:00 AM traveling along straight tracks at 80 and 90 mi/hr respectively. If the angle between their directions of travel is $118^{o,}$ to the nearest mile, how far are the trains from each other at 11:30 AM?

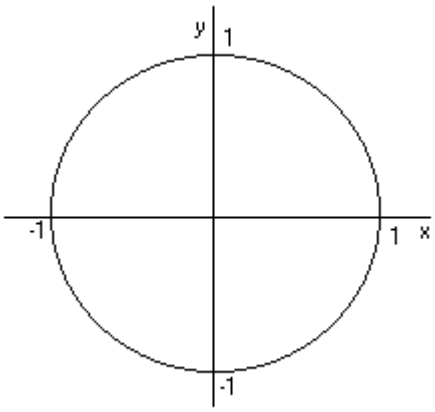
29. Given $t = 4$, complete the following:

a. Using $t = 4$, sketch on the unit circle the approximate location of $P(x,y)$, the terminal point

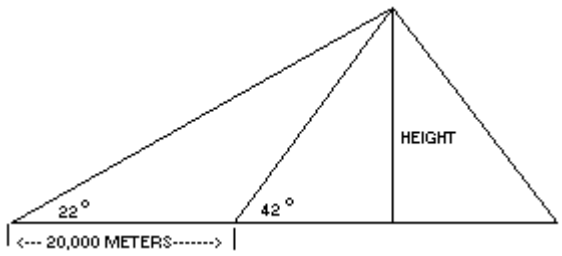
b. Find the reference number for t (to two decimal places)_____

c. What is the terminal point determined by t ?

(Give to two decimal places) P = (,)



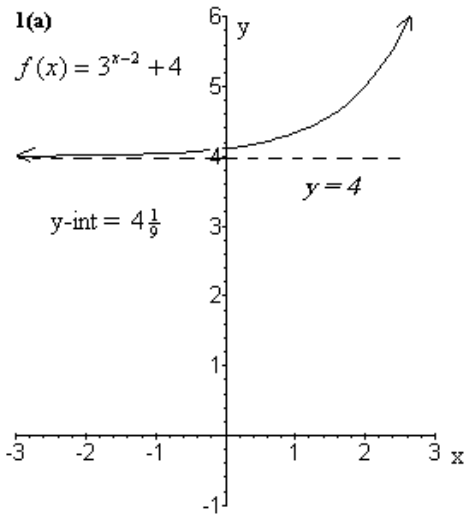
30. The top of a volcano is viewed from a safe distance of 20,000 meters level to the base of the volcano. The angle of inclination is found to be 22 degrees. If the angle of incline from the base of the volcano to its summit is found to be 42 degrees, to the nearest foot, how high is the volcano?



31. A pilot in an airplane flying at 25,000 ft sees two towns directly ahead of her in a straight line. The angles of the depression to the towns are 25^{o} and 50^{o}, respectively. To the nearest foot, how far apart are the towns?

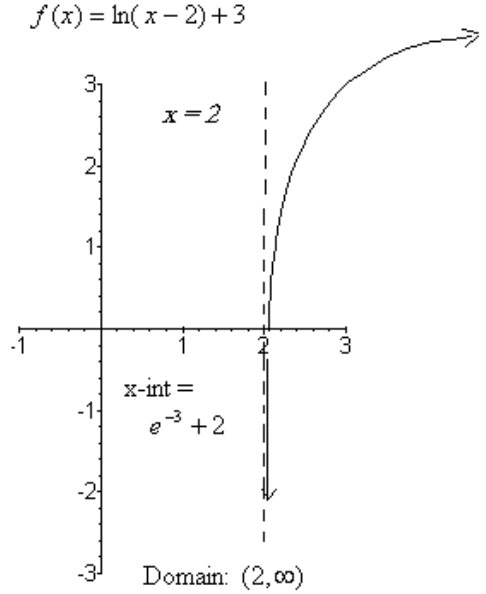
ANSWERS: MATH 112 FINAL EXAM REVIEW EXERCISES

1(a)



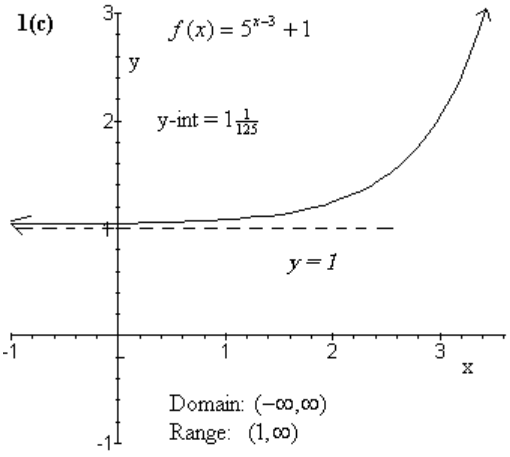
Domain: $(-\infty, \infty)$
Range: $(4, \infty)$

1(b) $f(x) = \ln(x-2) + 3$



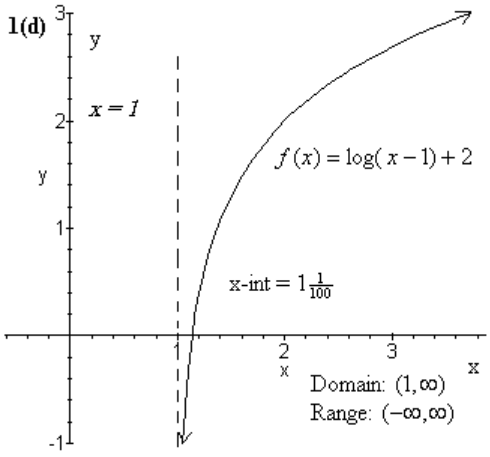
Domain: $(2, \infty)$
Range: $(-\infty, \infty)$

1(c)



Domain: $(-\infty, \infty)$
Range: $(1, \infty)$

1(d)



Domain: $(1, \infty)$
Range: $(-\infty, \infty)$

2.

X	15	9	$\frac{5}{3}$	5a	$\frac{3}{a^2}$
$\log_a x$	3.9	3.2	0.7	3.3	-0.4

3. If $\log_3 28 = x \Rightarrow 3^x = 28$. Hence $x > 3$. On the other hand, $\log_4 63 = y \Rightarrow 4^y = 63$. Hence $y < 3$. Therefore $\log_3 28 > \log_4 64$.

4. (a) $x = \dfrac{2}{999}$ **(b)** $x = 5$ **(c)** $x = \dfrac{1}{7}$ **(d)** $x = -12$ **(e)** $x = \pm\sqrt{2}$

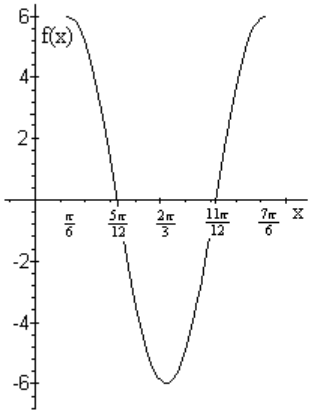
5. (a) (i) $x = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$ **(ii)** $x = \dfrac{\pi}{3} + 2\pi n, \dfrac{2\pi}{3} + 2\pi n$, n an integer.

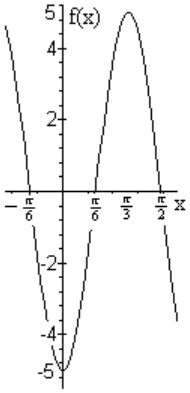
(b) (i) $x = \dfrac{\pi}{2}$ **(ii)** $x = \dfrac{\pi}{2} + 2\pi n$, n an integer.

(c) (i) $x = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$ **(ii)** $x = \dfrac{\pi}{4} + 2\pi n, \dfrac{3\pi}{4} + 2\pi n, \dfrac{5\pi}{4} + 2\pi n, \dfrac{7\pi}{4} + 2\pi n$, n an integer.

6. $x \approx 0.23, 0.50$

7. (a) $f(x) = 6\cos\left(2x - \dfrac{\pi}{3}\right)$ **(b)** $f(x) = -5\sin\left(3x + \dfrac{\pi}{2}\right)$





Period = π
Amplitude = 6
Phase shift = $\frac{\pi}{6}$ right

Period = $\frac{2\pi}{3}$
Amplitude = 5
Phase shift = $\frac{\pi}{6}$ left

8. (a) $\dfrac{\sqrt{6} - \sqrt{2}}{4}$ **(b)** $\dfrac{\sqrt{6} - \sqrt{2}}{4}$ **9.** 45°, $-\dfrac{\sqrt{2}}{2}$

10. (a) $\sin t = -\dfrac{\sqrt{33}}{7}$, $\tan t = -\dfrac{\sqrt{33}}{4}$, $\cot t = -\dfrac{4}{\sqrt{33}}$, $\sec t = \dfrac{7}{4}$, $\csc t = -\dfrac{7}{\sqrt{33}}$

(b) $\cos t = \dfrac{\sqrt{55}}{8}$, $\tan t = -\dfrac{3}{\sqrt{55}}$, $\cot t = -\dfrac{\sqrt{55}}{3}$, $\sec t = \dfrac{8}{\sqrt{55}}$, $\csc t = -\dfrac{8}{3}$

11. $\sin t = \dfrac{5}{\sqrt{41}}$, $\cos t = -\dfrac{4}{\sqrt{41}}$, $\tan t = -\dfrac{5}{4}$, $\cot = -\dfrac{4}{5}$, $\sec t = -\dfrac{\sqrt{41}}{4}$, $\csc t = \dfrac{\sqrt{41}}{5}$

12. (a) $\frac{7}{25}$ **(b)** $-\frac{4\sqrt{21}}{25}$ **(c)** $\frac{4\sqrt{2}-\sqrt{5}}{9}$

13. (a) $\frac{1-\sin^2 x}{\cot^2 x}=\frac{\cos^2 x}{\cot^2 x}=\frac{\cos^2 x}{\frac{\cos^2 x}{\sin^2 x}}=\cos^2 x\cdot\frac{\sin^2 x}{\cos^2 x}=\sin^2 x$

(b) $\frac{\cos x}{1+\sin x}+\frac{\cos x}{1-\sin x}=\frac{(1-\sin x)\cos x}{(1-\sin x)(1+\sin x)}+\frac{(1+\sin x)\cos x}{(1-\sin x)(1+\sin x)}=$

$$\frac{(1-\sin x)\cos x+(1+\sin x)\cos x}{(1-\sin x)(1+\sin x)}=\frac{\cos x-\sin x\cos x+\cos x+\sin x\cos x}{(1-\sin x)(1+\sin x)}=$$

$$\frac{2\cos x}{(1-\sin x)(1+\sin x)}=\frac{2\cos x}{1-\sin^2 x}=\frac{2\cos x}{\cos^2 x}=\frac{2}{\cos x}=2\sec x$$

14. (a) -30^o **(b)** 150^o **15. (a)** $\frac{\sqrt{5}}{3}$ **(b)** $-\frac{6}{\sqrt{37}}$ **16.** $\frac{x}{\sqrt{1-x^2}}$

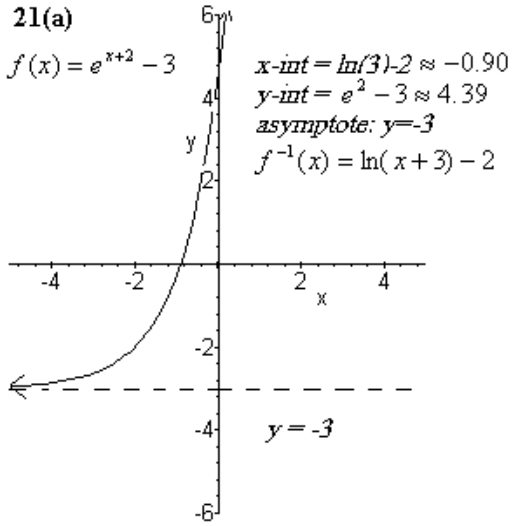
17. (a) $y(t)=2000e^{0.05t}$ **(b)** $g(t)=2500\left(1+\frac{0.03}{12}\right)^{12t}$ **(c)** *y(9)* = \$3,136.62 and *g(9)*=\$3,273.81 Hence John's account was worth more. **(d)** t = 11.1 years

18. (a) 6/7 **(b)** 4/7 **19. (a)** 2.05 **(b)** 2.46 **20. (a)** $f^{-1}(x)=\log_3 x$ **(b)** $f^{-1}(x)=3^x$

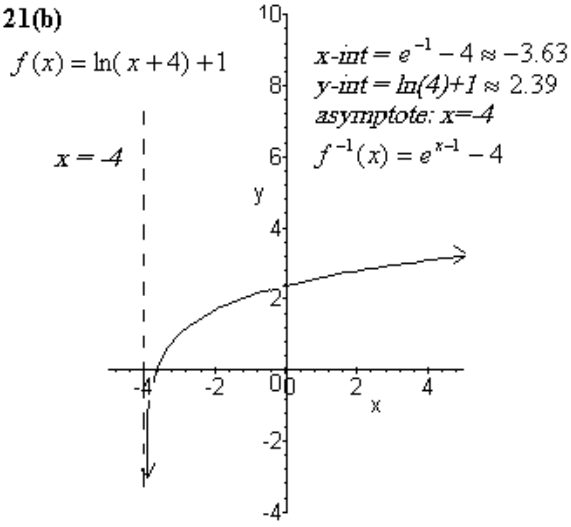
(c) $f^{-1}(x)=\log(x+3)-1$ **(d)** $f^{-1}(x)=10^{x-5}+4$ **(e)** $f^{-1}(x)=\frac{\ln(x+4)-1}{2}$

(f) $f^{-1}(x)=\frac{e^{x+5}-3}{2}$

21(a)



21(b)

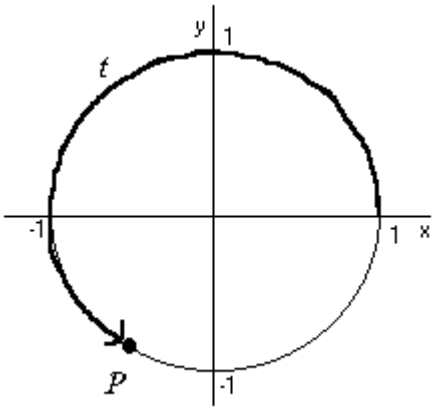


22.(a) $r \approx 0.041$ **(b)** 2012 **23.** 12,670 years old **24. (a)** 75^{o} **(b)** 49^{o}

(c) $62+\frac{13\sqrt{3}}{2}$ degrees **(d)** 68.5^{o} **(e)** 24 hours **25.** $a=\sqrt{10}, b=-5$

26. 13.61^{o} **27.** 5.24 inches **28.** 219 miles

29. (a) **(b)** 0.86 **(c)** *P=(-0.65,-0.76)*



30. 14,658 meters **31.** 32,635 feet