(10) 1. Suppose \( f(x) \) is a differentiable function with \( f(8) = 5 \), \( f'(8) = 3 \) and \( f''(8) = -2 \). If \( F(x) = f(x^3) \), compute \( F(2) \), \( F'(2) \), and \( F''(2) \).

\[
F(2) = \quad F'(2) = \quad F''(2) =
\]

(12) 2. Suppose \( f(x) = \frac{\ln(x^2 + 5)}{x + 1} \).

a) What is the domain of \( f(x) \)? Give some justification for your answer.
b) Compute \( \lim_{x \to +\infty} f(x) \).
c) \( y = f(x) \) has a horizontal asymptote. Use the answer to b) to write the equation of a horizontal asymptote to the curve \( y = f(x) \).

\[\text{ANSWER: } \]

d) Compute \( \lim_{x \to -1^-} f(x) \).
e) \( y = f(x) \) has a vertical asymptote. Use the answer to c) to write the equation of a vertical asymptote to the curve \( y = f(x) \).

\[\text{ANSWER: } \]

(16) 3. Suppose \( f(x) = (x^2 - 3)e^x \).
a) Find the first coordinates (the \( x \) values) of all relative maxima and minima of the function \( f(x) = (x^2 - 3)e^x \). Briefly explain your answers using calculus.
b) Find the first coordinates (the \( x \) values) of all inflection points of the function \( f(x) = (x^2 - 3)e^x \). Briefly explain your answers using calculus.

(16) 4. A farmer has 400 feet of fencing and wants to enclose a rectangular pasture. One side of the pasture is along a river, and does not need fencing. What should the dimensions of the rectangular pasture be, so as to maximize the enclosed area?

(14) 5. The program Maple displays the image shown to the right when asked to graph the equation \( y^2 = x^3 - 3xy + 3 \).
a) Verify by substitution that the point \( P = (-2, 1) \) is on the graph of the equation.
b) Find \( \frac{dy}{dx} \) in terms of \( y \) and \( x \). c) Find an equation for the line tangent to the graph at the point \( P = (-2, 1) \).
d) Sketch this tangent line in the appropriate place on the image displayed.
(12) Two circles have the same center. The inner circle has radius $r$ which is increasing at the rate of 3 inches per second. The outer circle has radius $R$ which is increasing at the rate of 2 inches per second. Suppose that $A$ is the area of the region between the circles. At a certain time, $r$ is 7 inches and $R$ is 10 inches. What is $A$ at that time? How fast is $A$ changing at that time? Is $A$ increasing or decreasing at that time?

(20) The graph of $y = f'(x)$, the derivative of the function $f(x)$, is shown to the right. Use the graph to answer the questions below.

The parts of this problem are not related but both parts use information from the graph of the derivative of $f'(x)$.

a) Use information from the graph of $f'(x)$ to find (as well as possible) the $x$ where the maximum value of $f(x)$ in the interval $1 \leq x \leq 3$ must occur. Briefly explain using calculus why your answer is correct, including verification that the value of $f(x)$ you select is larger than $f(x)$ at any other number in the interval.

b) Suppose that $f(3) = 5$. Use information from the graph and the tangent line approximation for $f(x)$ to find an approximate value of $f(3.04)$. Briefly explain using calculus and information from the graph why your approximate value for $f(3.04)$ is greater than or less than the exact value of $f(3.04)$. 

The graph of $f'(x)$, the derivative of $f(x)$
Second Exam for Math 135, section F2

August 3, 2006

NAME ________________________________

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No notes other than the distributed formula sheet may be used on this exam.
No calculators may be used on this exam.

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